



WINTER – 19 EXAMINATION

Subject Name: STRENGTH OF MATERIALS

Model Answer

22306

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.No.	Answer	Marking Scheme																
1.		<p>Attempt any <b>FIVE</b> of the following:</p> <p>a) Write the formulae to find moment of Inertia of semi-circular section about its xx and yy centroidal axes.</p> <p>Sol<sup>n</sup>:-</p> $I_{xx} = 0.11 R^4$ $I_{yy} = \frac{\pi R^4}{8} \text{ OR } \frac{\pi D^4}{128}$	10																
		<p>b) Differentiate between single shear and double shear.</p> <p>Sol<sup>n</sup>:- Differentiation between single shear and double shear</p> <table border="1"> <thead> <tr> <th>Sr No.</th> <th>Criteria</th> <th>Single shear</th> <th>Double shear</th> </tr> </thead> <tbody> <tr> <td>1.</td> <td>No. of shearing planes</td> <td>one</td> <td>two</td> </tr> <tr> <td>2.</td> <td>No. of pieces of specimen after failure in shear</td> <td>two</td> <td>three</td> </tr> <tr> <td>3.</td> <td>formula to calculate shear stress</td> <td><math>q = F/A</math></td> <td><math>q = F/2A</math></td> </tr> </tbody> </table>	Sr No.	Criteria	Single shear	Double shear	1.	No. of shearing planes	one	two	2.	No. of pieces of specimen after failure in shear	two	three	3.	formula to calculate shear stress	$q = F/A$	$q = F/2A$	<p>1. } 1. } 2</p> <p>1 Mark for each criteria Max. 2 Marks.</p>
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c) Define brittleness. Enlist any two names of brittle materials.

Brittleness:- It is the property of material due to which it suddenly breaks without remarkable deformation when subjected to external force.

Examples of brittle materials -

Brass, Cast iron, Glass, chalk etc.

01  
02  
1/2 mark for each example

d) Define point of contraflexure.

Point of contraflexure:- It is the point along the length of beam where bending moment changes from sagging (or +ve) to hogging (or -ve) and vice-versa.

02

e) State the relation between maximum shear stress and average shear stress for a solid circular section.

For solid circular beam section -

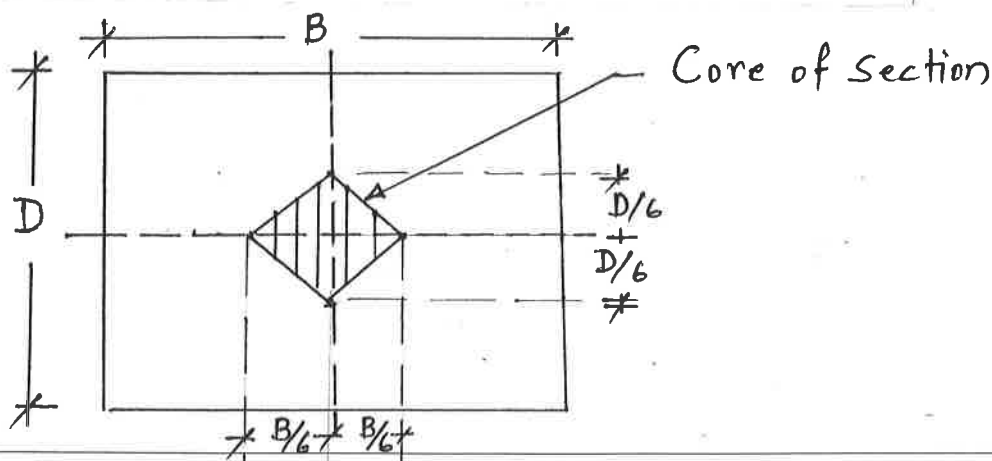
$$\text{Max. shear stress} = \frac{4}{3} \text{ Average shear stress}$$

OR.

$$\tau_{\text{max}} = \frac{4}{3} \tau_{\text{average}} \text{ or } 1.33 \tau_{\text{average}}$$

02

f) Draw a neat sketch to show core of a rectangular section of (B × D) dimensions.





1. g) State the condition for no tension at the base of a column.

Condition for no tension at the base of a column:-  
Direct stress shall be greater than or equal to bending stress

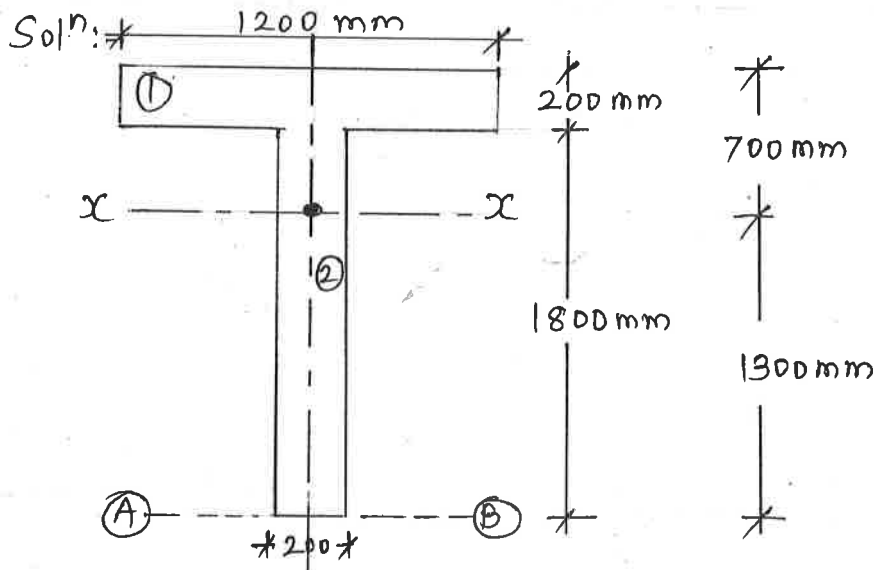
$$\sigma_o \geq \sigma_b \quad \text{OR}$$

02

2. Attempt any THREE of the following:

12

a) Calculate M.I. of a T-section about the centroidal xx axis. Top flange is 1200 mm × 200 mm and web is 1800 mm × 200 mm. Total height is 2000 mm.



from fig:-

$$a_1 = 1200 \times 200 = 240000 \text{ mm}^2, \quad y_1 = 1900 \text{ mm}$$

$$a_2 = 200 \times 1800 = 360000 \text{ mm}^2, \quad y_2 = 900 \text{ mm}$$

Position of centroid from base AB

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{240000(1900) + 360000(900)}{240000 + 360000}$$

$$\bar{y} = 1300 \text{ mm} \quad \text{OR} \quad 700 \text{ mm from top of flange.}$$

Using transfer formula,

$$I_{xx_1} = \left( \frac{bd^3}{12} + ah^2 \right)_1 = \frac{1200 \times 200^3}{12} + 240000(1900-1900)^2$$

$$I_{xx_1} = 8.72 \times 10^{10} \text{ mm}^4$$

$$I_{xx_2} = \left( \frac{bd^3}{12} + ah^2 \right)_2 = \frac{200 \times 1800^3}{12} + 360000(1300-900)^2$$



(Contd... from page 3)

$$I_{xx_2} = 15.48 \times 10^{10} \text{ mm}^4$$

$$\therefore I_{xx} \text{ of 'T' section} = I_{xx_1} + I_{xx_2} \\ = 8.72 \times 10^{10} + 15.48 \times 10^{10}$$

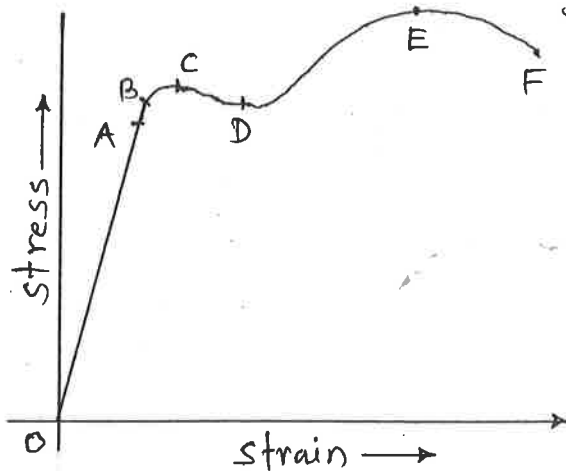
$$I_{xx} = 24.20 \times 10^{10} \text{ mm}^4$$

1

1

- 2 b) Draw stress - strain diagram with all salient points on it for ductile material and explain the term ultimate stress.

Sol<sup>n</sup>:- stress-strain diagram for ductile material



- A = Proportionality point
- B = Elastic limit
- C = Upper Yield point
- D = Lower Yield point
- E = Ultimate stress point
- F = Breaking stress point

1+1

Ultimate Stress:- It is maximum stress developed in material. Its value is obtained by following relation:

$$\text{Ultimate stress} = \frac{\text{Maximum load}}{\text{Original c/s area of body}}$$

02

- 2 c) For a certain material, modulus of elasticity is 169 MPa. If Poisson's ratio is 0.32, calculate the values of modulus of rigidity and bulk modulus.

Given:-  $E = 169 \text{ MPa}$   
 $\mu = 0.32$

- To find:-  
i) Modulus of rigidity (G)  
ii) Bulk Modulus (K)



Solution:-

i) Using the relation,  $E = 2G(1 + \mu)$   
 $169 = 2G(1 + 0.32)$

$\therefore G = 169 / 2.64 = 64.02 \text{ MPa}$

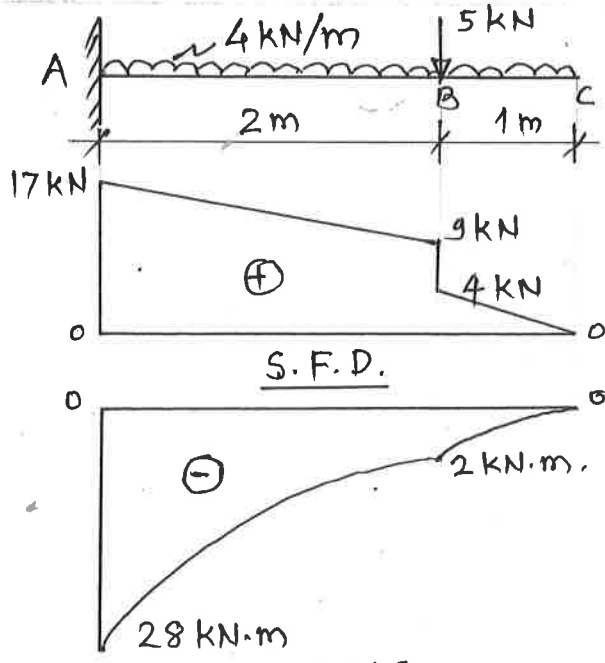
ii) Using the relation,  $E = 3K(1 - 2\mu)$

$169 = 3K(1 - 2 \times 0.32)$

$\therefore K = 169 / 1.08 = 156.48 \text{ MPa}$

2 d) A cantilever of span 3 m carries a point load of 5 kN at 2 m from the support and a u.d.l. of 4 kN/m over the entire span. Draw S.F. and B.M. diagrams.

Soln:-



$\uparrow \uparrow$  +ve.

$\curvearrowright \curvearrowright$  +ve.

S.F. Calculations,

$SF_c = 0$   
 $SF_B(\text{right}) = 4 \times 1 = 4 \text{ kN.}$   
 $SF_B(\text{left}) = 4 + 5 = 9 \text{ kN.}$   
 $SF_A = 9 + 4 \times 2 = 17 \text{ kN.}$

B.M. Calculations

$BM_c = 0$   
 $B.M_B = 4 \times 1 \times 0.5 = 2 \text{ kN.m.}$   
 $B.M_A = 4 \times 3 \times 1.5 + 5 \times 2 = 28 \text{ kN.m.}$



3.

Attempt any THREE of the following:

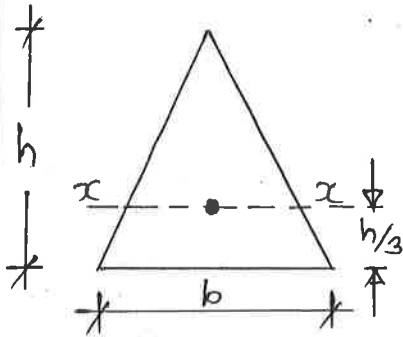
12

- a) State parallel axis theorem and use it to find moment of Inertia of an isosceles triangle of base 'b' and height 'h' about its base.

Sol<sup>n</sup>:- Parallel axis theorem:- Moment of inertia of a plane lamina about an axis parallel to its centroidal axis is given by sum of M.I. of that lamina about its centroidal axis and the product of area of the lamina and square of the distance between two parallel axes.

02

M.I. of an isosceles triangle about its base



Using parallel axis theorem,

$$\begin{aligned} I_{\text{base}} &= I_G + Ay^2 \\ &= \frac{bh^3}{36} + \left(\frac{1}{2} \times b \times h\right) \times \left(\frac{h}{3}\right)^2 \\ &= \frac{bh^3}{36} + \frac{bh^3}{18} \\ &= \frac{bh^3 + 2bh^3}{36} \end{aligned}$$

$$I_{\text{base}} = \frac{bh^3}{12}$$

02



3

- b) A brass bar shown in Figure No. 1 is subjected to a tensile load of 40 kN. Find the total elongation of the bar if  $E = 1 \times 10^5 \text{ N/mm}^2$  and the maximum stress induced.

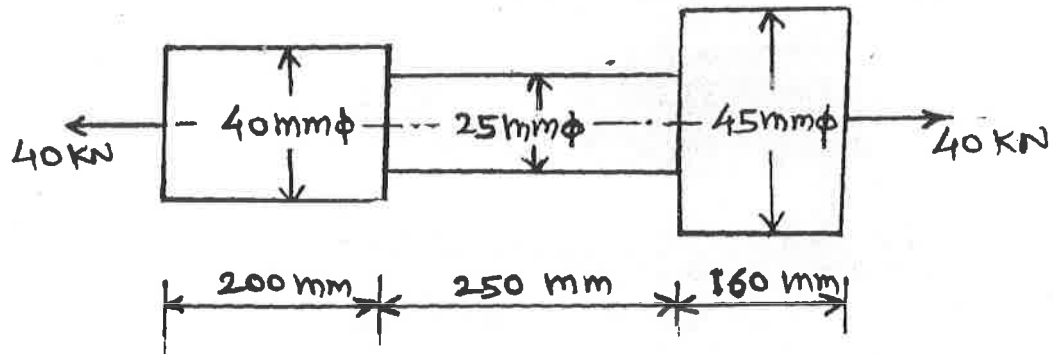


Fig. No. 1

To find:- (i) total elongation, (ii) maximum stress

Soln:-  $a_1 = \frac{\pi}{4} \times 40^2 = 1256.63 \text{ mm}^2$

$$a_2 = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$$

$$a_3 = \frac{\pi}{4} \times 45^2 = 1590.43 \text{ mm}^2$$

Change in length =  $\delta L = \frac{PL}{AE}$

$$\delta L = \frac{P}{E} \left( \frac{L_1}{a_1} + \frac{L_2}{a_2} + \frac{L_3}{a_3} \right)$$

$$= \frac{40 \times 10^3}{1 \times 10^5} \left[ \frac{200}{1256.63} + \frac{250}{490.87} + \frac{160}{1590.43} \right]$$

$$\delta L = 0.308 \text{ mm}$$

$$\text{Maximum stress} = \frac{P}{a_{\min}} = \frac{40 \times 10^3}{490.87}$$

$$\sigma_{\max} = 81.49 \text{ N/mm}^2$$

01

02

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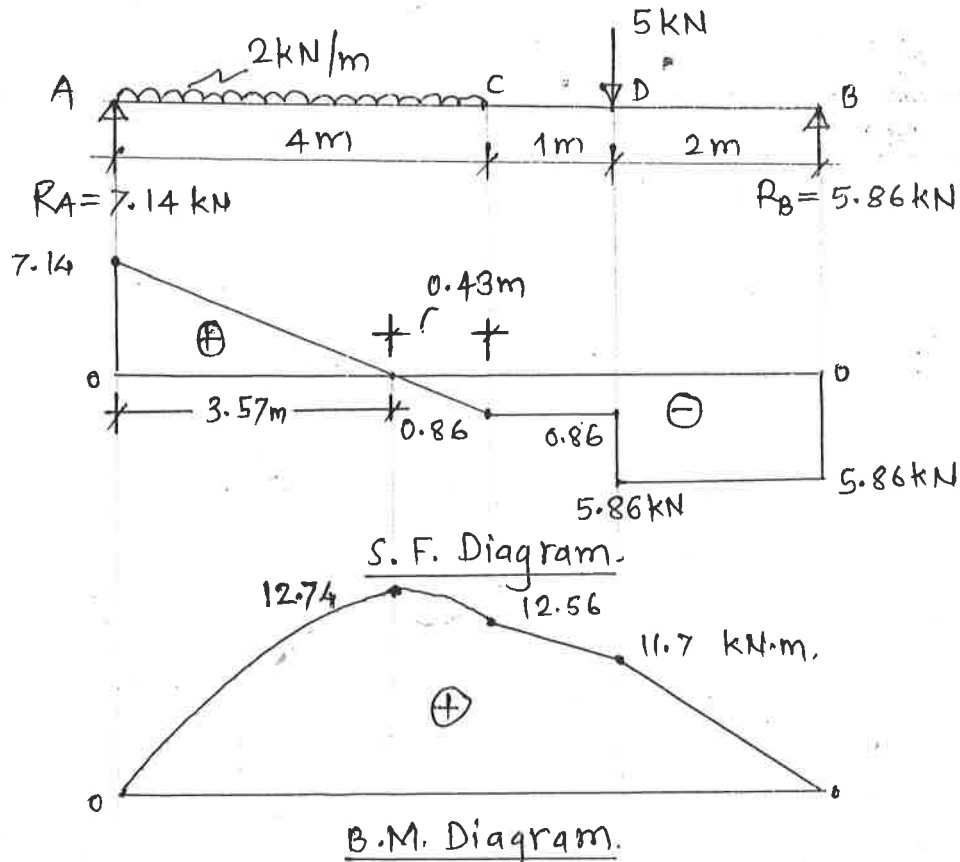


3

c)

A simply supported beam of span 7 m carries an u.d.l. of 2 kN/m over 4 m length from left hand support and a point load of 5 kN at 2 m from right hand support. Draw S.F. and B.M. diagrams.

Soln:-



01

01

Ⓐ Reactions,  $\sum M_A = 0$ ,  $\curvearrowright +ve$ .

$$2 \times 4 \times 2 + 5 \times 5 - R_B \times 7 = 0$$
$$\therefore R_B = 5.86 \text{ kN}$$

$\sum F_y = 0$   $\uparrow +ve$

$$R_A + R_B - (2 \times 4) - 5 = 0$$
$$\therefore R_A = 13 - 5.86 = 7.14 \text{ kN}$$

Ⓑ S.F. values  $\uparrow +ve$ .

$$S.F_A = R_A = 7.14 \text{ kN.}$$

$$S.F_D = 7.14 - (2 \times 4) = -0.86 \text{ kN.}$$

$$S.F_D (\text{left}) = -0.86 \text{ kN}$$

$$S.F_D (\text{right}) = -0.86 - 5 = -5.86 \text{ kN.}$$

$$S.F_B = -5.86 \text{ kN.}$$

S.F.  
Calculations

01  
Mark.





Position of point of contra-shear

$$\frac{7.14}{0.86} = \frac{4-d}{d} = \frac{4}{d} - 1$$

$$8.30 + 1 = \frac{4}{d} \quad \therefore d = \frac{4}{9.30} = 0.43 \text{ m. from pt. 'c'}$$

OR 3.57 m from 'A'

⊙ B.M. Values  $\left(\begin{array}{c} \uparrow \\ | \\ \uparrow \end{array}\right)$  +ve

$$BMA = 0$$

$$BM_c = 7.14 \times 4 - 2 \times 4 \times 2 = 12.56 \text{ kN}\cdot\text{m.}$$

$$BM_D = 7.14 \times 5 - 2 \times 4 \times 3 = 11.7 \text{ kN}\cdot\text{m.}$$

$$BMB = 0$$

$$BM_{\max} = 7.14 \times 3.57 - \frac{2 \times 3.57^2}{2} = 12.74 \text{ kN}\cdot\text{m.}$$

B.M.  
Calculation

01  
Mark.

3

- d) A C-clamp as shown in Figure No. 2, carries a load  $P = 25 \text{ kN}$ . The cross-section of the clamp at  $x-x$  is rectangular, having width equal to twice the thickness. Assuming that the C-clamp is made of steel casting with an allowable stress of  $100 \text{ N/mm}^2$ , find its dimensions  $b$  and  $t$ .

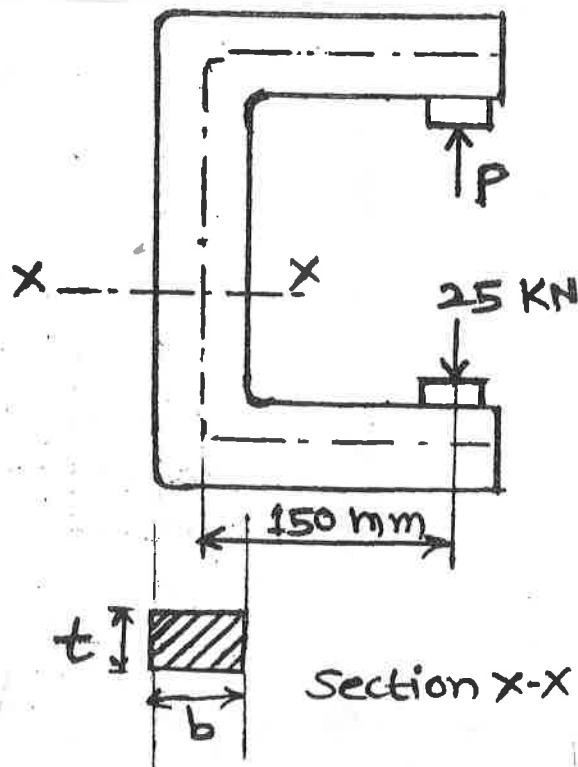


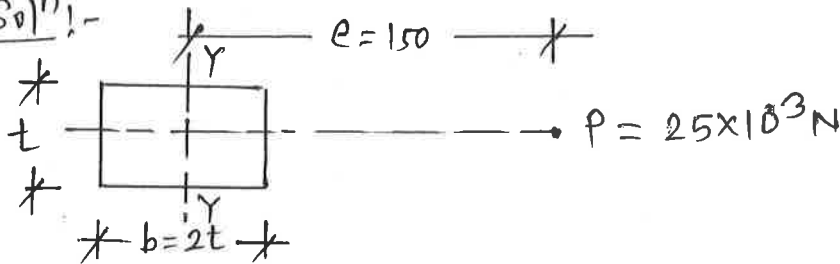
Fig. No. 2



d Given:  $P = 25 \times 10^3 \text{ N}$ ,  $e = 150 \text{ mm}$ ,  $\sigma_{\text{max}} = 100 \text{ N/mm}^2$   
 $b = 2t$

To find:  $b$  and  $t$

Soln:-



eccentricity @  $yy$ -axis.

$$I_{yy} = \frac{t \times (2t)^3}{12} = \frac{2}{3} t^4, \quad y_{\text{max}} = \frac{2t}{2} = t$$

$$\sigma_{\text{max}} = \sigma_0 + \sigma_b = \frac{P}{A} + \frac{P \cdot e \cdot y_{\text{max}}}{I}$$

$$100 = \frac{25 \times 10^3}{t \times 2t} + \frac{25 \times 10^3 \times 150 \times t}{\frac{2}{3} t^4}$$

$$100 = \frac{12500}{t^2} + \frac{5625000}{t^3}$$

OR multiplying by  $\frac{t^3}{100}$

$$t^3 = 125t + 56250$$

$$\therefore t^3 - 125t - 56250 = 0$$

$$\text{Solving, } \underline{t = 39.41 \text{ mm}}$$

$$\therefore b = 2t = 2 \times 39.41$$

$$\therefore \underline{b = 78.82 \text{ mm}}$$

01

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01

4.

Attempt any THREE of the following:

12

- a) A steel rail is 12.6m long and is laid at a temperature of 24°C. The maximum temperature expected is 44°C.

Determine:

- (i) The minimum gap between two rails to be left so that temperature stresses do not develop.
- (ii) Thermal stresses developed in the rails if no expansion joint is provided.

Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ .

Given:-  $L = 12.6 \text{ m} = 12.6 \times 10^3 \text{ m}$ ,  $t_1 = 24^\circ\text{C}$ ,  $t_2 = 44^\circ\text{C}$   
 $\alpha = 12.6 \times 10^{-6} / ^\circ\text{C}$ ,  $E = 2 \times 10^5 \text{ N/mm}^2$

To find:- i) Minimum gap so that no temp. stress develop  
 ii) thermal stresses developed if no expansion is permitted.

Soln:-  $T = t_2 - t_1 = 44 - 24 = 20^\circ\text{C}$

i) Minimum gap for no temp. stresses =  $\delta L$

$$\delta L = \text{free expansion} = L \alpha T$$

$$= 12.6 \times 10^3 \times 12 \times 10^{-6} \times 20$$

$$\delta L = 3.024 \text{ mm}$$

01+01

ii) Thermal stresses when no expansion joint is provided.

$$\sigma_T = \alpha T E = 12 \times 10^{-6} \times 20 \times 2 \times 10^5$$

$$\sigma_T = 48 \text{ N/mm}^2 \text{ (Compressive)}$$

01+01

4

- b) Calculate the power a shaft of 30 mm diameter can transmit with a speed of 200 r.p.m. if the permissible shear stress is  $120 \text{ N/mm}^2$ . Take maximum torque as 30% more than the average torque.

Given: for shaft,  $D = 30 \text{ mm}$ ,  $N = 200 \text{ rpm}$ ,  $\tau = 120 \text{ N/mm}^2$

$$T_{\max} = 1.3 T_{\text{avg}}$$

To find: Power of shaft

Sol<sup>n</sup>: Using the relation

$$\frac{T}{I_p} = \frac{\tau}{R}$$

$$\therefore T = \frac{I_p}{R} \times \tau = \frac{\pi}{16} D^3 \times \tau$$

$$T_{\max} = \frac{\pi}{16} \times 30^3 \times 120 = 6.36 \times 10^5 \text{ N}\cdot\text{mm} \quad 01$$

$$\therefore T_{\text{avg}} = \frac{T_{\max}}{1.3} = \frac{6.36 \times 10^5}{1.3} = \frac{6.36 \times 10^5}{1.3}$$

$$= 4.89 \times 10^5 \text{ N}\cdot\text{mm} = 4.89 \times 10^2 \text{ N}\cdot\text{m} \quad 01$$

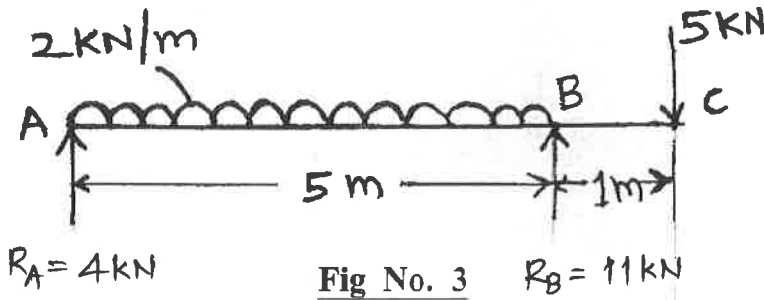
$$\therefore \text{Power} = \frac{2\pi N T_{\text{avg}}}{60} = \frac{2\pi \times 200 \times 4.89 \times 10^2}{60} \quad 01$$

$$= 10.242 \times 10^3 \frac{\text{N}\cdot\text{m}}{\text{sec}}$$

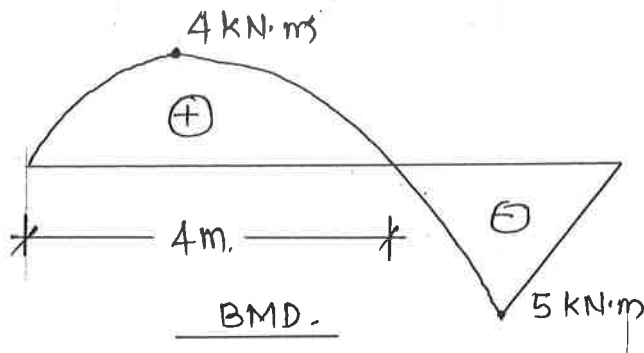
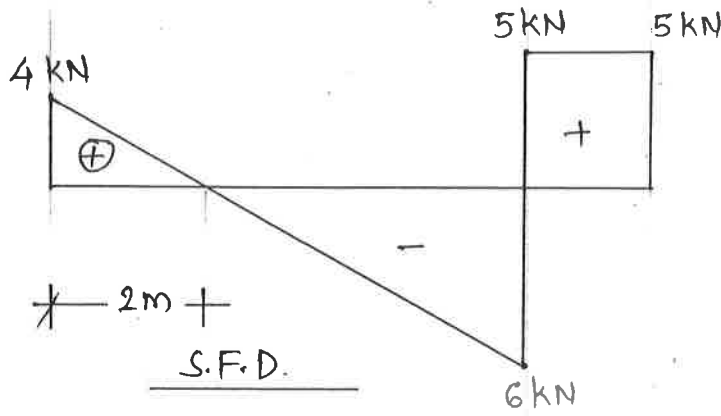
$$\boxed{P = 10.242 \text{ kW}} \quad 01$$

4

c) An overhanging beam is as shown in Figure No. 3. Draw S.F. and B.M. diagrams and locate the point of contraflexure.



Soln:-



1) Reactions

$$\sum M_A = 0 \quad \curvearrowright +ve.$$

$$2 \times 5 \times 2.5 - R_B \times 5 + 5 \times 6 = 0$$

$$\therefore R_B = 55/5 = 11 \text{ kN.}$$

$$\sum F_y = 0 \quad \uparrow +ve, \quad R_A + R_B - (2 \times 5) - 5 = 0$$

$$\therefore R_A = 15 - 11 = 4 \text{ kN.}$$

2) S.F. Values:-  $SF_A = 4 \text{ kN}$   $\uparrow +ve$

$$S.F._B(\text{left}) = 4 - (2 \times 5) = -6 \text{ kN.}$$

$$S.F._B(\text{right}) = -6 + 11 = 5 \text{ kN.}$$

$$S.F._C = 5 \text{ kN.}$$

S.F. Value  
& P.C. of  
Contraflexure  
01



Position of point of contraflexure.

$$\frac{4}{d} = \frac{6}{5-d} \rightarrow d = 2\text{m from 'A'}$$

3) B.M. Values  $\curvearrowright \uparrow$  +ve.

$$B.M_A = 0$$

$$B.M_B = 4 \times 5 - 2 \times 5 \times 2.5 = -5 \text{ kN}\cdot\text{m}$$

$$B.M_C = 0$$

$$B.M_{\text{max}} = 4 \times 2 - 2 \times 2 \times 1 = 4 \text{ kN}\cdot\text{m}$$

Position of point of contraflexure

$$BM_x = 0$$

$$4x - \frac{2x^2}{2} = 0$$

$$4x = x^2$$

$$\therefore x = 4\text{m from 'A'}$$

BM value  
and  
position  
of P.C.  
01

4 d) A simply supported beam of span 8m carries a point load of 60 kN at the centre of the span. Calculate the modulus of section required, if bending stress is not to exceed 150 MPa.

Given, for S.S. beam,  $L = 8\text{m}$ ,  $P = 60\text{ kN}$  at midspan.

$$\sigma_{b,\text{max}} = 150 \text{ MPa}$$

To find :- Section Modulus,  $Z$

Soln:-

$$\text{Maximum B.M at center} = \frac{WL}{4} = \frac{60 \times 8}{4} = 120 \text{ kN}\cdot\text{m}$$

$$\therefore M = 120 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\sigma_{b,\text{max}} = M/Z$$

$$\text{OR } Z = \frac{M}{\sigma_{b,\text{max}}} = \frac{120 \times 10^6}{150} = 8 \times 10^5 \text{ mm}^3$$

$$\boxed{Z = 8 \times 10^5 \text{ mm}^3}$$

01  
01  
01  
01

e) State the equation of torsion with the meaning of each term and use the torsional equation to find torque induced in a solid circular shaft of 50 mm diameter rotating at 100 r.p.m. The permissible shear stress is not to exceed 75 MPa.

Soln:-

i) Equation of Torsion

$$\frac{T}{I_p} = \frac{q}{R} = \frac{G\theta}{L}$$

where,  $T =$  Torque

$I_p =$  Polar M.I.

$q =$  Max. shear stress

$R =$  Radius of shaft

$G =$  Modulus of rigidity

$\theta =$  Angle of twist

$L =$  Length of shaft.

ii) for shaft,  $d = 50 \text{ mm}$ ,  $N = 100 \text{ rpm}$ ,  $q_{\text{max}} = 75 \text{ N/mm}^2$

To find :- Torque.

$$I_p = \frac{\pi d^4}{32} = \frac{\pi \times 50^4}{32} = 6.14 \times 10^5 \text{ mm}^4$$

$$T = \frac{I_p}{R} \times q_{\text{max}} = \frac{6.14 \times 10^5}{25} \times 75$$

$$\therefore T = 18.42 \times 10^5 \text{ N}\cdot\text{mm}$$

$$T = 1.842 \text{ kN}\cdot\text{m}$$



5.

Attempt any **TWO** of the following:

- a) A rectangular block loaded is shown in Figure No. 4. Find linear strains in X, Y and Z directions. Also find change in volume of the block. Take  $E = 200 \text{ GPa}$  and Poisson's ratio.  $\mu = 0.25$

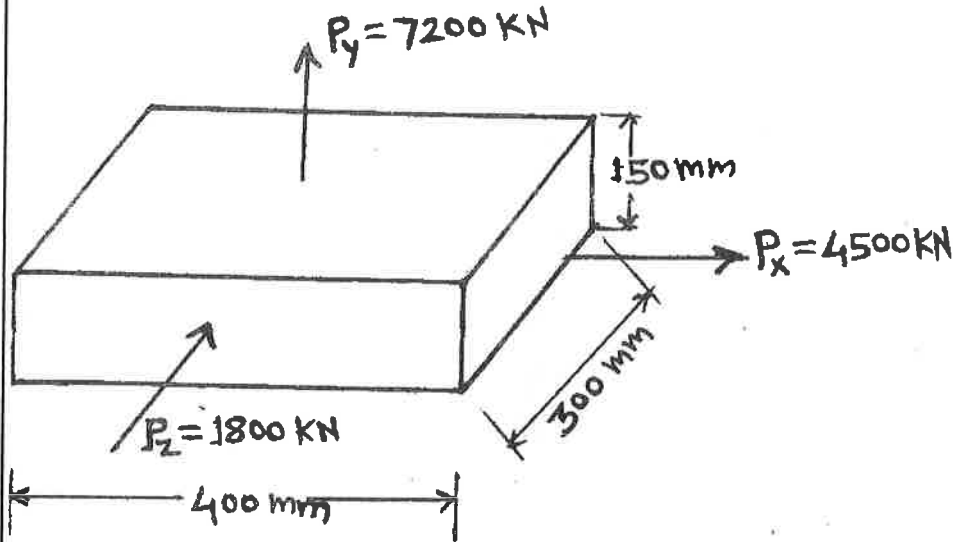


Fig No. 4

To find, i)  $e_x$  ii)  $e_y$  iii)  $e_z$ , iv)  $\delta V$ .

Solution:-

$$\sigma_z = \frac{4500 \times 10^3}{150 \times 300} = 100 \text{ N/mm}^2$$

$$\sigma_y = \frac{7200 \times 10^3}{400 \times 300} = 60 \text{ N/mm}^2$$

$$\sigma_z = \frac{-1800 \times 10^3}{400 \times 150} = -30 \text{ N/mm}^2$$

$$\begin{aligned} \text{Strain, } e_x &= \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} \\ &= \frac{1}{E} (\sigma_x - \mu \sigma_y - \mu \sigma_z) \\ &= \frac{1}{2 \times 10^5} (100 - 0.25 \times 60 - (-30) \times 0.25) \end{aligned}$$

$$e_x = 4.625 \times 10^{-4}$$

01

01





$$\text{Strain, } e_y = -\mu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu \cdot \frac{\sigma_z}{E}$$
$$= \frac{1}{E} (-0.25 \times 100 + 60 - 0.25 \times (-30))$$

$$e_y = +2.125 \times 10^{-4}$$

01

$$\text{Strain, } e_z = -\mu \frac{\sigma_x}{E} - \mu \cdot \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$
$$= \frac{1}{2 \times 10^5} (-0.25 \times 100 - 0.25 \times 60 + (-30))$$

$$e_z = -3.5 \times 10^{-4}$$

01

$$\therefore \text{Volumetric strain} = e_v = e_x + e_y + e_z$$
$$= 4.625 \times 10^{-4} + 2.125 \times 10^{-4} - 3.5 \times 10^{-4}$$

$$\therefore e_v = 3.25 \times 10^{-4}$$

01

$$\text{Change in volume} = e_v \times V$$

$$= +3.25 \times 10^{-4} \times (400 \times 300 \times 150)$$

$$\delta V = +5850 \text{ mm}^3$$

01

5

b) A simply supported beam of span 8 m carries two point loads of 50 kN and 20 kN at 2 m and 6 m from the left hand support respectively. Draw bending moment diagram and also sketch the qualitative deflected shape of the beam.

Soln:- Reactions of beam.

$$\sum M_A = 0 \quad \curvearrowright \text{ +ve.}, \quad 50 \times 2 + 20 \times 6 - R_B \times 8 = 0$$

$$\therefore R_B = 27.50 \text{ kN.}$$

$$\sum F_y = 0 \quad \uparrow \text{ +ve.}, \quad R_A + R_B - 50 - 20 = 0$$

$$\therefore R_A = 70 - 27.50 = 42.50 \text{ kN.}$$

Calculations for B.M.D.  $(\curvearrowright) \text{ +ve.}$

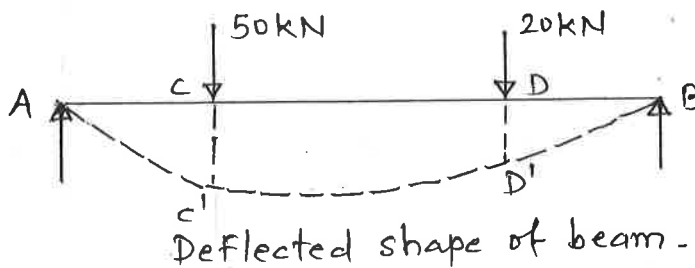
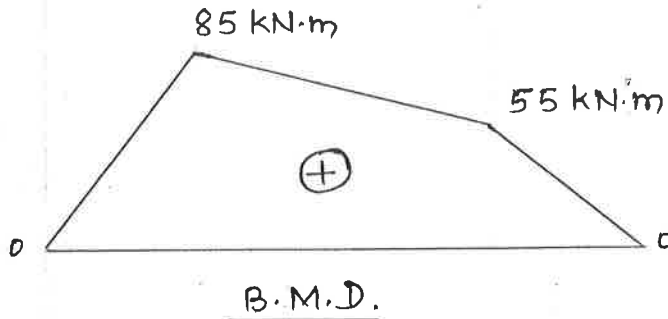
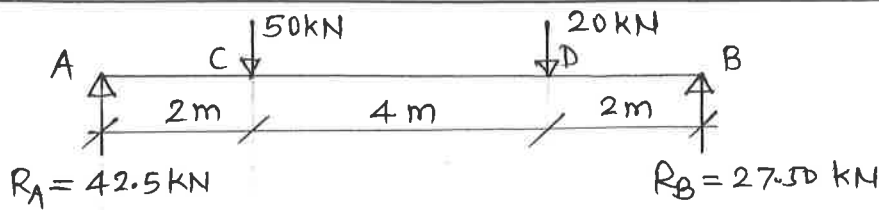
$$BMA = MB = 0 \quad \text{--- simple supports.}$$

$$BM_B = R_A \times 2 = 42.50 \times 2 = 85 \text{ kN.m.}$$

$$BM_D = 42.5 \times 6 - 50 \times 4 = 55 \text{ kN.m.}$$

01

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5

c) State any four assumptions in the theory of simple bending and apply the bending stress equation to determine the maximum bending stress developed in a rectangular beam of cross section  $50 \text{ mm} \times 150 \text{ mm}$  when a bending moment of  $600 \text{ N}\cdot\text{m}$  is applied about x-x axis.

Sol<sup>n</sup>:- Assumptions in the theory of simple bending

1. The section which is plane before bending will remain plane after bending.
2. The material of beam is homogeneous & isotropic.
3. Every layer is free to expand or contract as compared to the layer above or below it.
4. The beam is loaded within the elastic limit.
5. The section is subjected to pure bending only.

for rectangular beam,  $b = 50 \text{ mm}$ ,  $d = 150 \text{ mm}$ ,  $M = 600 \times 10^3 \text{ N}\cdot\text{mm}$

$$I_{xx} = \frac{bd^3}{12} = \frac{50 \times 150^3}{12} = 14.06 \times 10^6 \text{ mm}^4$$

$$y_{\text{max}} = d/2 = 150/2 = 75 \text{ mm.}$$

1/2 mark  
for each  
assumption  
(max. 2 Mk)

01

01



Applying bending equation,

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\therefore \sigma_{b,max} = \pm \frac{M \cdot y_{max}}{I} = \pm \frac{600 \times 10^3 \times 75}{14.06 \times 10^6}$$

$$\therefore \sigma_{b,max} = \pm 3.20 \text{ N/mm}^2$$

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6.

Attempt any TWO of the following:

12

- a) A beam of square cross section  $100 \text{ mm} \times 100 \text{ mm}$  is subjected to a shear force of  $30 \text{ kN}$ . Calculate the maximum shear stress as well as shear stress induced across the section at a layer  $20 \text{ mm}$  away from the neutral axis. Sketch the shear stress distribution diagram for the given beam.

Given :- for beam section

$$b = d = 100 \text{ mm.}$$

$$S = 30 \times 10^3 \text{ N.}$$

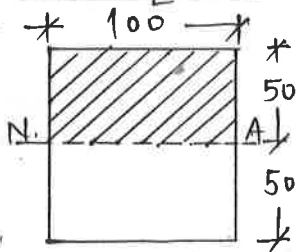
To find:

- i) Max. shear stress ( $q_{max}$ )
- ii) Shear stress at a layer  $20 \text{ mm}$  away from N.A.
- iii) Shear stress distribution

Soln:-

Shear stress is given by:-  $q = \frac{S a \bar{y}}{b I}$

i) for  $q_{max}$ .



$$S = 30 \times 10^3 \text{ N}$$

$$A = 100 \times 50 = 5000 \text{ mm}^2$$

$$\bar{y} = 50/2 = 25 \text{ mm}$$

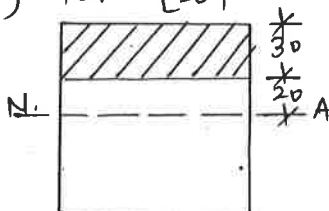
$$b = 100 \text{ mm, } I_{xx} = \frac{100 \times 100^3}{12} = 8.33 \times 10^6 \text{ mm}^4$$

$$\therefore q_{max} = \frac{S a \bar{y}}{b I} = \frac{30 \times 10^3 \times 5000 \times 25}{100 \times 8.33 \times 10^6}$$

$$q_{max} = 4.5 \text{ N/mm}^2$$

03

ii) for  $q_{20}$



$$S = 30 \times 10^3 \text{ N, } A = 100 \times 30 = 3000 \text{ mm}^2$$

$$\bar{y} = 20 + \frac{30}{2} = 35 \text{ mm.}$$

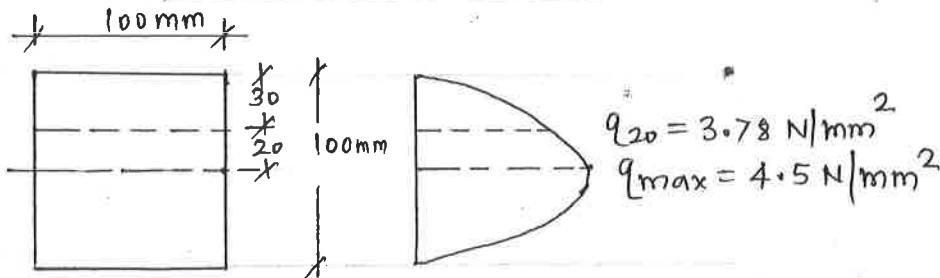
$$b = 100 \text{ mm.}$$

$$I = 8.33 \times 10^6 \text{ mm}^4$$



$$q_{20} = \frac{S a \bar{y}}{b I} = \frac{30 \times 10^3 \times 3000 \times 35}{100 \times 8.93 \times 10^6}$$

$$\therefore \boxed{q_{20} = 3.78 \text{ N/mm}^2}$$



C/S of beam

Shear stress distribution

02

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6

b)

A hollow circular shaft is required to transmit a torque of 24 kN.m. The inside diameter is 0.6 times external diameter. Calculate both the diameter, if allowable shear stress is 80 MPa.

Given, for hollow circular shaft

External dia =  $D$

Internal dia =  $d = 0.6D$

$\tau_s = 80 \text{ N/mm}^2$ ,  $T = 24 \times 10^6 \text{ N}\cdot\text{mm}$

Solution:-

$$I_p = \frac{\pi}{32} [D^4 - d^4] = \frac{\pi}{32} [D^4 - (0.6D)^4] = 0.0855D^4$$

$$R = D/2$$

Using the relation,  $\frac{T}{I_p} = \frac{\tau_s}{R}$

$$\frac{24 \times 10^6}{0.0855D^4} = \frac{80 \times 2}{D}$$

$$\therefore D^3 = \frac{24 \times 10^6}{0.0855 \times 80 \times 2} = 1.75 \times 10^6 \text{ mm}^3$$

$$\therefore D = 120.61 \text{ mm, Say } 125 \text{ mm.}$$

$$d = 0.6D = 0.60 \times (125) = 75 \text{ mm.}$$

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6

c) A rectangular column 150 mm wide and 100 mm thick carries a load of 150 kN at an eccentricity of 50 mm in the plane bisecting the thickness. Find the maximum and minimum intensities of stress at the base section. Draw the combined stress distribution diagram showing these values.

Given, for rectangular column

$$b = 150 \text{ mm}, d = 100 \text{ mm.}$$

$$P = 150 \times 10^3 \text{ N}$$

$$e = 50 \text{ mm.}$$

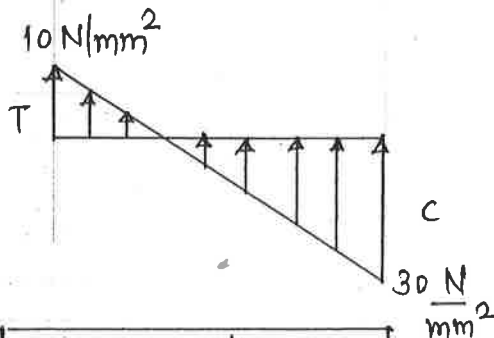
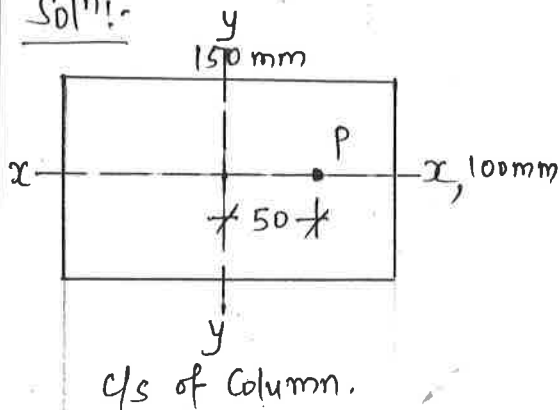
To find:-

1.)  $\sigma_{\text{max}}$

2.)  $\sigma_{\text{min}}$

3) stress distribution.

Soln:-



Combined stress distribution dia.

Eccentricity @ yy-axis -

$$A = 150 \times 100 = 15000 \text{ mm}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{150 \times 10^3}{15000} = 10 \text{ N/mm}^2 \quad 01$$

$$I_{yy} = \frac{100 \times 150^3}{12} = 28.13 \times 10^6 \text{ mm}^4 \quad 01$$

$$y_{\text{max}} = 150/2 = 75 \text{ mm.}$$

$$\sigma_b = \pm \frac{P \cdot e \cdot y_{\text{max}}}{I}$$

$$= \pm \frac{150 \times 10^3 \times 50 \times 75}{28.13 \times 10^6}$$

$$\sigma_b = \pm 20 \text{ N/mm}^2. \quad 01 \text{ mark for dia.}$$

$$\sigma_{\text{max}} = \sigma_0 + \sigma_b = 10 + 20$$

$$\therefore \sigma_{\text{max}} = 30 \text{ N/mm}^2 \text{ (Comp).} \quad 01$$

$$\sigma_{\text{min}} = \sigma_0 - \sigma_b = 10 - 20$$

$$\therefore \sigma_{\text{min}} = -10 \text{ N/mm}^2$$

$$\therefore \sigma_{\text{min}} = 10 \text{ N/mm}^2 \text{ (Tensile)} \quad 01$$