



WINTER- 19 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code: **22224**

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		Solve any <u>FIVE</u> of the following:	10
	a)	State whether the function is odd or even, $f(x) = \frac{e^x + e^{-x}}{2}$	02
	Ans	$f(x) = \frac{e^x + e^{-x}}{2}$ $\therefore f(-x) = \frac{e^{-x} + e^{-(-x)}}{2}$ $\therefore f(-x) = \frac{e^{-x} + e^x}{2}$ $\therefore f(-x) = f(x)$ $\therefore \text{function is even.}$ <p>-----</p>	 ½ ½ ½ ½
	b)	If $f(x) = \log_4 x + 3$, find $f\left(\frac{1}{4}\right)$	02
	Ans	$f(x) = \log_4 x + 3$ $f\left(\frac{1}{4}\right) = \log_4\left(\frac{1}{4}\right) + 3$ $= -\log_4 4 + 3$ $= -1 + 3 = 2$	 1 1



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1.	c)	Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$	02
	Ans	$y = x^2 \cdot e^x$ $\frac{dy}{dx} = x^2 \cdot e^x + e^x \cdot 2x$ $\frac{dy}{dx} = xe^x(x+2)$	1 1
	d)	Evaluate $\int [e^x + a^x + x^a + a^a] dx$	02
	Ans	$\int [e^x + a^x + x^a + a^a] dx$ $= e^x + \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$	2
e)	Evaluate: $\int \left[\frac{1}{1 + \cos 2x} \right] dx$	02	
Ans	$\int \left[\frac{1}{1 + \cos 2x} \right] dx$ $= \int \left[\frac{1}{2 \cos^2 x} \right] dx$ $= \frac{1}{2} \int \sec^2 x dx$ $= \frac{1}{2} \tan x + c$	1 1	
f)	Find the area bounded by $y = x$, X-axis and $x = 0$ to $x = 4$.	02	
Ans	Area $A = \int_a^b y dx$ $= \int_0^4 x dx$ $= \left[\frac{x^2}{2} \right]_0^4$	$\frac{1}{2}$ $\frac{1}{2}$	



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1.		$= \left(\frac{4^2}{2} - 0 \right)$ $= 8$	$\frac{1}{2}$ $\frac{1}{2}$
	g)	Find a real root of the equation $x^3 + 4x - 9 = 0$ in the interval (1, 2) by using Bisection method. (only one iteration)	02
	Ans	Let $f(x) = x^3 + 4x - 9$ $f(1) = -4$ $f(2) = 7$ \therefore the root is in (1, 2) $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$	1 1
2		Solve any THREE of the following:	12
	a)	Find $\frac{dy}{dx}$, if $y = \frac{5e^x}{3e^x + 1}$ at $x = 0$	04
	Ans	$y = \frac{5e^x}{3e^x + 1}$ $\frac{dy}{dx} = \frac{(3e^x + 1)5e^x - 5e^x(3e^x)}{(3e^x + 1)^2}$ $\frac{dy}{dx} = \frac{15e^{2x} + 5e^x - 15e^{2x}}{(3e^x + 1)^2}$ $\frac{dy}{dx} = \frac{5e^x}{(3e^x + 1)^2}$ at $x = 0$ $\frac{dy}{dx} = \frac{5e^0}{(3e^0 + 1)^2}$ $= \frac{5}{16} \text{ or } 0.3125$	2 1 1



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2.	b)	If $x = a(1 + \cos \theta)$, $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$	04
	Ans	$x = a(1 + \cos \theta)$, $y = a(1 - \cos \theta)$ $\frac{dx}{d\theta} = a(-\sin \theta) = -a \sin \theta$, $\frac{dy}{d\theta} = a(0 + \sin \theta) = a \sin \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{-a \sin \theta}$ $\frac{dy}{dx} = -1$	1+1 1 1
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	04
	Ans	Let length of rectangle = x , breadth = y $\therefore 2x + 2y = 36$ $\therefore y = 18 - x$ Area $A = x \times y$ $A = x(18 - x)$ $\therefore A = 18x - x^2$ $\therefore \frac{dA}{dx} = 18 - 2x$ $\therefore \frac{d^2A}{dx^2} = -2$ Let $\frac{dA}{dx} = 0$ $\therefore 18 - 2x = 0$ $\therefore x = 9$ at $x = 9$ $\frac{d^2A}{dx^2} = -2 < 0$ Area is maximum at $x = 9$ Length = 9 ; breadth = 9	1 1 1/2 1/2 1/2 1/2



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3.	a)	$\therefore \frac{dy}{dx} = \frac{-2}{9}$ $\therefore \text{slope of tangent, } m = \frac{-2}{9}$ <p>Equation of tangent at (1,2) is</p> $y - 2 = \frac{-2}{9}(x - 1)$ $\therefore 9y - 18 = -2x + 2$ $\therefore 2x + 9y - 20 = 0$ $\therefore \text{slope of normal, } m' = \frac{-1}{m} = \frac{9}{2}$ <p>Equation of normal at (1,2) is</p> $y - 2 = \frac{9}{2}(x - 1)$ $\therefore 2y - 4 = 9x - 9$ $\therefore 9x - 2y - 5 = 0$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	b)	<p>Find $\frac{dy}{dx}$ if $y = \tan^{-1} \left[\frac{2x}{1+35x^2} \right]$</p>	04
	Ans	$y = \tan^{-1} \left[\frac{7x - 5x}{1 + 7x \cdot 5x} \right]$ $y = \tan^{-1} 7x - \tan^{-1} 5x$ $\frac{dy}{dx} = \frac{7}{1+49x^2} - \frac{5}{1+25x^2}$	<p>1</p> <p>1</p> <p>2</p>
c)	<p>If $x^y = e^{x-y}$ Show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$</p>	04	
Ans	$x^y = e^{x-y}$ $\log x^y = \log e^{x-y}$ $y \log x = x - y \log e$ $y \log x = x - y$ $y \log x + y = x$	<p>1/2</p> <p>1/2</p> <p>1/2</p>	



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3.	c)	$y(\log x + 1) = x$ $y = \frac{x}{\log x + 1}$ $\frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \cdot \frac{1}{x}}{(\log x + 1)^2}$ $= \frac{\log x}{(\log x + 1)^2}$	1 1 ½
	d)	<p>Evaluate $\int \frac{dx}{5 + 3 \cos 2x}$</p> <p>Ans $\int \frac{dx}{5 + 3 \cos 2x}$</p> <p>Put $\tan x = t$, $dx = \frac{dt}{1+t^2}$</p> $\cos 2x = \frac{1-t^2}{1+t^2}$ $\int \frac{\frac{dt}{1+t^2}}{5 + 3 \left(\frac{1-t^2}{1+t^2} \right)}$ $= \int \frac{dt}{5(1+t^2) + 3(1-t^2)}$ $= \int \frac{dt}{5 + 5t^2 + 3 - 3t^2}$ $= \int \frac{dt}{2t^2 + 8}$ $= \int \frac{dt}{(\sqrt{2}t)^2 + (\sqrt{8})^2} \quad \text{OR} \quad = \frac{1}{2} \int \frac{dt}{t^2 + 4}$ $= \frac{1}{\sqrt{8}} \tan^{-1} \left(\frac{\sqrt{2}t}{\sqrt{8}} \right) \cdot \frac{1}{\sqrt{2}} + c \quad = \frac{1}{2} \cdot \frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) + c$	04 1 1 1



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3.	d)	$= \frac{1}{\sqrt{16}} \tan^{-1} \left(\frac{\sqrt{2} \tan x}{\sqrt{8}} \right) + c \quad \text{OR} \quad = \frac{1}{4} \tan^{-1} \left(\frac{t}{2} \right) + c$ $= \frac{1}{4} \tan^{-1} \left(\frac{\tan x}{2} \right) + c \quad \text{OR} \quad = \frac{1}{4} \tan^{-1} \left(\frac{\tan x}{2} \right) + c$	<p>½</p> <p>½</p>
4.		<p>Solve any <u>THREE</u> of the following:</p>	12
	a)	<p>Evaluate $\int \frac{[e^x(x+1)]}{\cos^2(x.e^x)} dx$</p>	04
	Ans	$\int \frac{[e^x(x+1)]}{\cos^2(x.e^x)} dx$ <p>Put $x.e^x = t$ $\therefore (x.e^x + e^x.1) dx = dt$ $[e^x(x+1)] dx = dt$ $\therefore \int \frac{1}{\cos^2 t} dt$ $= \int \sec^2 t dt$ $= \tan t + c$ $= \tan(x.e^x) + c$</p>	<p>1</p> <p>1</p> <p>1</p> <p>½</p> <p>½</p>
	b)	<p>Evaluate: $\int \frac{dx}{2x^2 + 3x + 2}$</p>	04
	Ans	$\int \frac{dx}{2x^2 + 3x + 2}$ $= \frac{1}{2} \int \frac{dx}{x^2 + \frac{3}{2}x + 1}$ $= \frac{1}{2} \int \frac{dx}{x^2 + \frac{3}{2}x + \frac{9}{16} + 1 - \frac{9}{16}}$	<p>½</p> <p>1</p>



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4.	b)	$= \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{4}\right)^2 + \frac{7}{16}}$ $= \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$ $= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{7}}{4}} \tan^{-1} \left(\frac{x + \frac{3}{4}}{\frac{\sqrt{7}}{4}} \right) + c$ $= \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4x + 3}{\sqrt{7}} \right) + c$	<p>½</p> <p>1</p> <p>1</p>
	c)	<p>-----</p> <p>Evaluate $\int x^2 \cdot \tan x \, dx$</p> <p>Ans $\int x^2 \cdot \tan x \, dx$</p> $= x^2 \left(\int \tan x \, dx \right) - \int \left(\int \tan x \, dx \cdot \frac{d}{dx}(x^2) \right) dx$ $= x^2 \log(\sec x) - \int \log(\sec x) \cdot 2x \, dx$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \int \frac{1}{\sec x} \cdot \sec x \tan x \cdot \frac{x^2}{2} dx \right]$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \frac{1}{2} \int x^2 \cdot \tan x \, dx \right]$ $= x^2 \log(\sec x) - 2 \left[\log(\sec x) \frac{x^2}{2} - \frac{1}{2} I \right]$ $I = x^2 \log(\sec x) - \log(\sec x) x^2 + I$	<p>04</p> <p>½</p> <p>1</p> <p>½</p> <p>½</p> <p>½</p>
	d)	<p>-----</p> <p>Evaluate $\int \frac{\sec^2 x}{(\tan x)(\tan x + 1)} dx$</p>	<p>04</p>

Note: If students attempted to solve the question give appropriate marks.



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4.	Ans	$\int \frac{\sec^2 x}{(\tan x)(\tan x + 1)} dx$ <p>Put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $\therefore \int \frac{1}{(t)(t+1)} dt$ $\frac{1}{(t)(t+1)} = \frac{A}{t} + \frac{B}{t+1}$ $\therefore 1 = A(t+1) + B(t)$ \therefore Put $t = 0$, $A = 1$ Put $t = -1$, $B = -1$ $\therefore \frac{1}{(t)(t+1)} = \frac{1}{t} - \frac{1}{t+1}$ $\therefore \int \frac{1}{(t)(t+1)} dt = \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$ $= \log(t) - \log(t+1) + c$ $= \log(\tan x) - \log(\tan x + 1) + c$</p>	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
	e) Ans	<p>Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$</p> $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$ $= \int_0^{\frac{\pi}{2}} \frac{1}{1 + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} dx$ <p>Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \dots \dots \dots (1)$</p>	<p>04</p>



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4.	e)	$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right) + \sqrt{\sin\left(\frac{\pi}{2}-x\right)}}} dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \dots\dots\dots (2)$ <p>Add (1) and (2)</p> $I+I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$ $2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$ $2I = \int_0^{\frac{\pi}{2}} dx$ $2I = [x]_0^{\frac{\pi}{2}}$ $2I = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$	<p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>
5	a) Ans	<p>Solve any TWO of the following:</p> <p>Find area bounded by the curve $y = x^2$ and the line $y = x$</p> <p>We have $y = x^2$ and $y = x$</p> $\therefore x^2 - x = 0$ $\therefore x(x-1) = 0$ $\therefore x = 0 \text{ or } x = 1$ $\text{Area} = \int_a^b (y_1 - y_2) dx$ $= \int_0^1 (x^2 - x) dx$	<p>12</p> <p>06</p> <p>1</p> <p>1</p>



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5.	a)	$= \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1$ $= \left[\frac{1^3}{3} - \frac{1^2}{2} - 0 \right]$ $= -\frac{1}{6}$ <p>$\therefore A = \frac{1}{6}$ or 0.167 (\because Area is always +ve)</p>	1 1 1 1
	b)	Attempt the following:	06
	i)	From the differential equation by eliminating the arbitrary constant if	03
	Ans	$y = A \cos x + B \sin x.$ $y = A \cos x + B \sin x.$ $\frac{dy}{dx} = -A \sin x + B \cos x$ $\frac{d^2y}{dx^2} = -A \cos x - B \sin x$ $= -(A \cos x + B \sin x)$ $= -y$ $\frac{d^2y}{dx^2} + y = 0$	1 1 1
	ii)	Solve $(1+x^2)dy - x^2.ydx = 0$	03
	Ans	$(1+x^2)dy - x^2.ydx = 0$ $(1+x^2)dy = x^2.ydx$ $\frac{dy}{y} = \frac{x^2 dx}{1+x^2}$ $\int \frac{dy}{y} = \int \frac{x^2 dx}{1+x^2}$ $\int \frac{dy}{y} = \int \frac{1+x^2 - 1 dx}{1+x^2}$	1



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5.		$\int \frac{dy}{y} = \int \left[1 - \frac{1}{1+x^2} \right] dx$ $\log y = x - \tan^{-1} x + c$	1 1
	c) Ans	<p>Solve the D.E $\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$ given that $q = 0$ when $t = 0$ and E,R,C are constant</p> $\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$ $I.F = e^{\int \frac{1}{RC} dt}$ $= e^{\frac{t}{RC}}$ $\therefore q.e^{\frac{t}{RC}} = \int \frac{E}{R}.e^{\frac{t}{RC}} dt$ $= \frac{E}{R} e^{\frac{t}{RC}} \cdot \frac{1}{\frac{1}{RC}} + c_1$ $q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC + c_1$ <p>given that $q = 0$ when $t = 0$</p> $0 = e^0 EC + c_1$ $c_1 = -EC$ $q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC - EC$ $q = EC \left(1 - e^{-\frac{t}{RC}} \right)$	06 1 1 1 1 1
6.		<p>Solve any TWO of the following:</p>	12
	a) i)	<p>Attempt the following:</p> <p>Solve the equations by Gauss-Seidal method. (two iterations only)</p> $10x + y + 2z = 13, \quad 3x + 10y + z = 14, \quad 2x + 3y + 10z = 15$	06 03



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6.	c)	<p>Initial root $x_0=2$ $\therefore f'(2)=5$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.8$ $x_2 = 1.8 - \frac{f(1.8)}{f'(1.8)} = 1.7913$ $x_3 = 1.7913 - \frac{f(1.7913)}{f'(1.7913)} = 1.7912$ $x_4 = 1.7912 - \frac{f(1.7912)}{f'(1.7912)} = 1.7912$ OR Let $f(x) = x^2 + x - 5$ $f(1) = -3 < 0$ $f(2) = 1 > 0$ $f'(x) = 2x + 1$ Initial root $x_0=2$ $x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 + x - 5}{2x + 1}$ $= \frac{2x^2 + x - x^2 - x + 5}{2x + 1}$ $= \frac{x^2 + 5}{2x + 1}$ $x_1 = 1.8$ $x_2 = 1.7913$ $x_3 = 1.7912$ $x_4 = 1.7912$</p>	<p>1 1 1 1 1 1 2 ½ ½ ½ ½</p>
		<p><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	