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WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics　　　**Model Answer**　　　**Subject Code:** 22210

Important Instructions to Examiners:
1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	If $f(x) = \tan x$, show that $f(2x) = \dfrac{2f(x)}{1-\left[f(x)\right]^2}$	**02**
	Ans	$f(2x)$	
		$= \tan 2x$	½
		$= \dfrac{2\tan x}{1-\tan^2 x}$	1
		$= \dfrac{2f(x)}{1-\left[f(x)\right]^2}$	½
		OR	
		$\dfrac{2f(x)}{1-\left[f(x)\right]^2}$	
		$= \dfrac{2\tan x}{1-\tan^2 x}$	½
		$= \tan 2x$	1
		$= f(2x)$	½
		---	
	b)	State whether the function $f(x) = \dfrac{e^x + e^{-x}}{2}$ is even or odd.	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b) Ans	$f(x) = \dfrac{e^x + e^{-x}}{2}$	
		$\therefore f(-x) = \dfrac{e^{-x} + e^{-(-x)}}{2}$	½
		$\therefore f(-x) = \dfrac{e^{-x} + e^x}{2}$	½
		$\therefore f(-x) = f(x)$	½
		$\therefore$ function is even.	½
		---	
	c)	Find $\dfrac{dy}{dx}$ if $y = x.e^x$	**02**
	Ans	$y = x.e^x$	
		$\therefore \dfrac{dy}{dx} = x\dfrac{d}{dx}\left(e^x\right) + e^x\dfrac{d}{dx}(x)$	
		$\therefore \dfrac{dy}{dx} = xe^x + e^x.1$	2
		$\therefore \dfrac{dy}{dx} = xe^x + e^x$	
		---	
	d)	Evaluate $\int \tan^{-1} x\, dx$	**02**
	Ans	$\int \tan^{-1} x\, dx = \int \tan^{-1} x\, .1\, dx$	
		$= \tan^{-1} x\int 1dx - \int\left(\left(\int 1dx\right)\dfrac{d}{dx}\left(\tan^{-1} x\right)\right)dx$	½
		$= x\tan^{-1} x - \int x\dfrac{1}{1+x^2}dx$	½
		$= x\tan^{-1} x - \int \dfrac{x}{1+x^2}dx$	
		$= x\tan^{-1} x - \dfrac{1}{2}\int \dfrac{2x}{1+x^2}dx$	½
		$= x\tan^{-1} x - \dfrac{1}{2}\log\left(1+x^2\right)+c$	½
		---	
	e)	Evaluate $\int \sqrt{1+\sin 2x}\, dx$	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	e)	$\int \sqrt{1+\sin 2x} \ dx$	
	Ans	$= \int \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} \ dx$	½
		$= \int \sqrt{(\sin x + \cos x)^2} \ dx$	½
		$= \int (\sin x + \cos x) \ dx$	½
		$= -\cos x + \sin x + c$	½
	f)	Find the area bounded by the curve $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$	**02**
	Ans	Area $A = \int_{a}^{b} y \ dx$	
		$= \int_{0}^{\pi} \sin x \ dx$	½
		$= [-\cos x]_{0}^{\pi}$	½
		$= [-\cos \pi] - [-\cos 0]$	½
		$= -(-1) - (-1)$	
		$= 2$	½
	g)	Express in the form $a + ib$, $Z = \dfrac{1+i}{2-i}$, where $a, b, \in \mathrm{IR}$. $i = \sqrt{-1}$	**02**
	Ans	$Z = \dfrac{1+i}{2-i}$	
		$= \dfrac{1+i}{2-i} \times \dfrac{2+i}{2+i}$	½
		$= \dfrac{2+i+2i+i^2}{2^2 - i^2}$	½
		$= \dfrac{2+3i-1}{4+1}$	½
		$= \dfrac{1+3i}{5}$	
		$= \dfrac{1}{5} + \dfrac{3i}{5}$	½

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Q. No.	Sub Q. N.	Answers	Marking Scheme
2.		**Attempt any THREE of the following:**	**12**
	a)	If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\dfrac{dy}{dx}$	**04**
	Ans	$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$	
		$x = a(\theta - \sin\theta)$	
		$\therefore \dfrac{dx}{d\theta} = a(1 - \cos\theta)$	1
		$y = a(1 - \cos\theta)$	
		$\therefore \dfrac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta$	1
		$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \dfrac{a\sin\theta}{a(1 - \cos\theta)}$	1
		$\dfrac{dy}{dx} = \dfrac{\sin\theta}{1 - \cos\theta}$	1
	b)	If $x^2 + y^2 = xy$ find $\dfrac{dy}{dx}$	**04**
	Ans	$x^2 + y^2 = xy$	
		$\therefore 2x + 2y\dfrac{dy}{dx} = x\dfrac{dy}{dx} + y.1$	1
		$\therefore 2x + 2y\dfrac{dy}{dx} = x\dfrac{dy}{dx} + y$	1
		$\therefore (2y - x)\dfrac{dy}{dx} = y - 2x$	1
		$\therefore \dfrac{dy}{dx} = \dfrac{y - 2x}{2y - x}$	1
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle $= x$, breadth $= y$	
		$\therefore 2x + 2y = 36$	
		$\therefore y = 18 - x$	1
		Area $A = x \times y$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	$A = x(18-x)$	
		$\therefore A = 18x - x^2$	1
		$\therefore \dfrac{dA}{dx} = 18 - 2x$	½
		$\therefore \dfrac{d^2 A}{dx^2} = -2$	½
		Let $\dfrac{dA}{dx} = 0$	
		$\therefore 18 - 2x = 0$	
		$\therefore x = 9$	½
		at $x = 9$	
		$\dfrac{d^2 A}{dx^2} = -2 < 0$	
		Area is maximum at $x = 9$	
		Length $= 9$; breadth $= 9$	½
		--	
	d)	A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$. Find the radius of curvature of the beam at this point at $x = \dfrac{\pi}{2}$	**04**
	Ans	$y = 2\sin x - \sin 2x$	
		$\therefore \dfrac{dy}{dx} = 2\cos x - 2\cos 2x$	½
		$\therefore \dfrac{d^2 y}{dx^2} = -2\sin x + 4\sin 2x$	½
		at $x = \dfrac{\pi}{2}$	
		$\therefore \dfrac{dy}{dx} = 2\cos\left(\dfrac{\pi}{2}\right) - 2\cos 2\left(\dfrac{\pi}{2}\right) = 2$	½
		$\therefore \dfrac{d^2 y}{dx^2} = -2\sin\left(\dfrac{\pi}{2}\right) + 4\sin 2\left(\dfrac{\pi}{2}\right) = -2$	½
		$\therefore$ Radius of curvature $= \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2 y}{dx^2}} = \dfrac{\left[1 + (2)^2\right]^{\frac{3}{2}}}{-2}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore$ Radius of curvature $= -5.59$ $= 5.59$	1

3.		**Attempt any THREE of the following:**	**12**
	a)	Find the equation of the tangent and normal to the curve $4x^2 + 9y^2 = 40$ at $(1,2)$	**04**
	Ans	$4x^2 + 9y^2 = 40$	
		$\therefore 8x + 18y\dfrac{dy}{dx} = 0$	½
		$\therefore \dfrac{dy}{dx} = \dfrac{-8x}{18y}$	
		$\therefore \dfrac{dy}{dx} = \dfrac{-4x}{9y}$	½
		at $(1,2)$	
		$\therefore \dfrac{dy}{dx} = \dfrac{-4(1)}{9(2)}$	
		$\therefore \dfrac{dy}{dx} = \dfrac{-2}{9}$	
		$\therefore$ slope of tangent , $m = \dfrac{-2}{9}$	½
		Equation of tangent at $(1,2)$ is	
		$y - 2 = \dfrac{-2}{9}(x-1)$	½
		$\therefore 9y - 18 = -2x + 2$	
		$\therefore 2x + 9y - 20 = 0$	½
		$\therefore$ slope of normal , $m' = \dfrac{-1}{m} = \dfrac{9}{2}$	½
		Equation of normal at $(1,2)$ is	
		$y - 2 = \dfrac{9}{2}(x-1)$	½
		$\therefore 2y - 4 = 9x - 9$	
		$\therefore 9x - 2y - 5 = 0$	½

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	Find $\dfrac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$	**04**
	Ans	Let $u = x^{\sin x}$	
		$\therefore \log u = \log x^{\sin x}$	
		$\therefore \log u = \sin x \log x$	½
		$\therefore \dfrac{1}{u}\dfrac{du}{dx} = \sin x \cdot \dfrac{1}{x} + \log x . \cos x$	1
		$\therefore \dfrac{1}{u}\dfrac{du}{dx} = \dfrac{\sin x}{x} + \cos x \log x$	
		$\therefore \dfrac{du}{dx} = u\left(\dfrac{\sin x}{x} + \cos x \log x\right)$	
		$\therefore \dfrac{du}{dx} = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \log x\right)$	½
		Let $v = (\tan x)^x$	
		$\therefore \log v = \log(\tan x)^x$	
		$\therefore \log v = x \log(\tan x)$	½
		$\therefore \dfrac{1}{v}\dfrac{dv}{dx} = x \cdot \dfrac{1}{\tan x} . \sec^2 x + \log(\tan x) \cdot 1$	1
		$\therefore \dfrac{1}{v}\dfrac{dv}{dx} = \dfrac{x \sec^2 x}{\tan x} + \log(\tan x)$	
		$\therefore \dfrac{dv}{dx} = v\left[\dfrac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	
		$\therefore \dfrac{dv}{dx} = (\tan x)^x\left[\dfrac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	½
		$\therefore \dfrac{dy}{dx} = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \log x\right) + (\tan x)^x\left[\dfrac{x \sec^2 x}{\tan x} + \log(\tan x)\right]$	
		--	
	c)	Find $\dfrac{dy}{dx}$ if $y = \log\left[x + \sqrt{x^2 + a^2}\right]$	**04**
	Ans	$y = \log\left[x + \sqrt{x^2 + a^2}\right]$	
		$\therefore \dfrac{dy}{dx} = \dfrac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \dfrac{1}{2\sqrt{x^2 + a^2}}(2x + 0)\right)$	2

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\therefore \dfrac{dy}{dx} = \dfrac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \dfrac{x}{\sqrt{x^2 + a^2}}\right)$ $\therefore \dfrac{dy}{dx} = \dfrac{1}{x + \sqrt{x^2 + a^2}}\left(\dfrac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2}}\right)$ $\therefore \dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 + a^2}}$	2
		--	
	d)	Evaluate $\displaystyle\int \dfrac{dx}{4 + 5\cos x}$	**04**
	Ans	$\displaystyle\int \dfrac{dx}{4 + 5\cos x}$ Put $\tan\dfrac{x}{2} = t$, $dx = \dfrac{2dt}{1 + t^2}$ $\cos x = \dfrac{1 - t^2}{1 + t^2}$	
		$\displaystyle\int \dfrac{\dfrac{2dt}{1 + t^2}}{4 + 5\left(\dfrac{1 - t^2}{1 + t^2}\right)}$	1
		$= \displaystyle\int \dfrac{2dt}{4(1 + t^2) + 5(1 - t^2)}$	½
		$= 2\displaystyle\int \dfrac{dt}{4 + 4t^2 + 5 - 5t^2}$	
		$= 2\displaystyle\int \dfrac{dt}{9 - t^2}$	1
		$= 2\displaystyle\int \dfrac{dt}{3^2 - t^2}$	
		$= 2\dfrac{1}{2 \cdot 3} \log\left|\dfrac{3 + t}{3 - t}\right| + c$	1
		$= \dfrac{1}{3} \log\left|\dfrac{3 + \tan\dfrac{x}{2}}{3 - \tan\dfrac{x}{2}}\right| + c$	½
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		**Attempt any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int \dfrac{(x-1)e^x}{x^2 \sin^2\left(\dfrac{e^x}{x}\right)}\, dx$	**04**
	Ans	$\int \dfrac{(x-1)e^x}{x^2 \sin^2\left(\dfrac{e^x}{x}\right)}\, dx$	
		Put $\dfrac{e^x}{x} = t$	
		$\therefore \dfrac{xe^x - e^x(1)}{x^2}\, dx = dt$	1
		$\therefore \dfrac{(x-1)e^x}{x^2}\, dx = dt$	
		$\therefore \int \dfrac{1}{\sin^2 t}\, dt$	½
		$= \int \cos ec^2 t\, dt$	1
		$= -\cot t + c$	1
		$= -\cot\left(\dfrac{e^x}{x}\right) + c$	½
		--	
	b)	Evaluate $\int \sin^3 x\, dx$	**04**
	Ans	$\int \sin^3 x\, dx$	
		since $\sin 3x = 3\sin x - 4\sin^3 x$ $\therefore \sin^3 x = \dfrac{1}{4}(3\sin x - \sin 3x)$	
		$\therefore \int \dfrac{1}{4}(3\sin x - \sin 3x)\, dx$	2
		$= \dfrac{1}{4}\left(-3\cos x + \dfrac{\cos 3x}{3}\right) + c$	2
		$OR \quad \int \sin^3 x\, dx$	½
		$= \int \sin^2 x\ \sin x\, dx$	
		$= \int (1 - \cos^2 x)\sin x\, dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	Put $\cos x = t$	
		$\therefore -\sin x \, dx = dt$	
		$\therefore \sin x \, dx = -dt$	1
		$\therefore \int (1 - t^2)(-dt)$	
		$= -\int (1 - t^2) \, dt$	
		$= -\left(t - \dfrac{t^3}{3} \right) + c$	1
		$= -\left(\cos x - \dfrac{\cos^3 x}{3} \right) + c$	½
		---	
	c)	Evaluate $\int \dfrac{2x^2 + 5}{(x-1)(x+2)(x+3)} \, dx$	**04**
	Ans	$\int \dfrac{2x^2 + 5}{(x-1)(x+2)(x+3)} \, dx$	
		Consider $\dfrac{2x^2 + 5}{(x-1)(x+2)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$	½
		$\therefore 2x^2 + 5 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$	
		Put $x = 1$	
		$\therefore 7 = 12A$	
		$\therefore A = \dfrac{7}{12}$	½
		Put $x = -2 \qquad \Rightarrow 13 = -3B$	
		$\therefore B = \dfrac{-13}{3}$	½
		Put $x = -3 \qquad \Rightarrow 23 = 4C$	
		$\therefore C = \dfrac{23}{4}$	½
		$\therefore \int \dfrac{2x^2 + 5}{(x-1)(x+2)(x+3)} \, dx = \int \left(\dfrac{\frac{7}{12}}{x-1} + \dfrac{\frac{-13}{3}}{x+2} + \dfrac{\frac{23}{4}}{x+3} \right) dx$	
		$= \dfrac{7}{12} \log(x-1) - \dfrac{13}{3} \log(x+2) + \dfrac{23}{4} \log(x+3) + c$	2

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	Evaluate $\int x^2 . e^{3x} \, dx$	**04**
	Ans	$\int x^2 . e^{3x} \, dx$	
		$= x^2 \left(\int e^{3x} dx \right) - \int \left(\int e^{3x} dx . \dfrac{d}{dx}(x^2) \right) dx$	1
		$= x^2 \dfrac{e^{3x}}{3} - \int \dfrac{e^{3x}}{3} . 2x \, dx$	1
		$= \dfrac{x^2 e^{3x}}{3} - \dfrac{2}{3} \int x e^{3x} \, dx$	
		$= \dfrac{x^2 e^{3x}}{3} - \dfrac{2}{3} \left[x \left(\int e^{3x} dx \right) - \int \left(\int e^{3x} dx . \dfrac{d}{dx}(x) \right) dx \right]$	½
		$= \dfrac{x^2 e^{3x}}{3} - \dfrac{2}{3} \left[x \dfrac{e^{3x}}{3} - \int \dfrac{e^{3x}}{3} . 1 \, dx \right]$	1
		$= \dfrac{x^2 e^{3x}}{3} - \dfrac{2}{3} \left[\dfrac{x e^{3x}}{3} - \dfrac{e^{3x}}{9} \right] + c$	½
		---	
	e)	Evaluate $\displaystyle\int_0^5 \dfrac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} \, dx$	**04**
	Ans	$I = \displaystyle\int_0^5 \dfrac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} \, dx \qquad \text{--------------------}(1)$	
		$I = \displaystyle\int_0^5 \dfrac{\sqrt{5-(5-x)}}{\sqrt{5-x}+\sqrt{5-(5-x)}} \, dx$	1
		$\therefore I = \displaystyle\int_0^5 \dfrac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} \, dx \qquad \text{--------------------}(2)$	
		add (1) and (2)	
		$I + I = \displaystyle\int_0^5 \dfrac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} \, dx + \int_0^5 \dfrac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} \, dx$	1
		$2I = \displaystyle\int_0^5 \dfrac{\sqrt{5-x}+\sqrt{x}}{\sqrt{x}+\sqrt{5-x}} \, dx$	
		$2I = \displaystyle\int_0^5 1 \, dx$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$2I = \left[x\right]_0^5$ $2I = 5 - 0$ $I = \dfrac{5}{2}$ ---	1 1
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the area of the circle $x^2 + y^2 = 36$ by using definite integration.	**06**
	Ans	$x^2 + y^2 = 36$ $\therefore y^2 = 36 - x^2$ $\therefore y = \sqrt{36 - x^2}$ $A = 4\displaystyle\int_0^6 \sqrt{36 - x^2}\,dx$ $= 4\displaystyle\int_0^6 \sqrt{6^2 - x^2}\,dx$ $= 4\left[\dfrac{x}{2}\sqrt{6^2 - x^2} + \dfrac{6^2}{2}\sin^{-1}\left(\dfrac{x}{6}\right)\right]_0^6$ $= 4\left[0 + \dfrac{36}{2}\sin^{-1}(1) - 0\right]$ $= 4\left[\dfrac{36}{2}\cdot\dfrac{\pi}{2}\right]$ $= 36\pi$ ---	1 2 1 1 1
	b) i)	Find the order and degree of D.E. $\sqrt{\dfrac{d^2 y}{dx^2}} - \dfrac{dy}{dx} - xy^2 = 0$	**03**
	Ans	$\sqrt{\dfrac{d^2 y}{dx^2}} = \dfrac{dy}{dx} + xy^2$ $\therefore \dfrac{d^2 y}{dx^2} = \left(\dfrac{dy}{dx} + xy^2\right)^2$ $\therefore$ Order $= 2$ Degree $= 1$	1 1 1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b) ii)	Solve D.E. $x \cdot \dfrac{dy}{dx} + y = x^3$	**03**
	Ans	$x \cdot \dfrac{dy}{dx} + y = x^3$	
		$\therefore \dfrac{dy}{dx} + \dfrac{y}{x} = x^2$	
		Comparing with $\dfrac{dy}{dx} + Py = Q$	
		$\therefore P = \dfrac{1}{x}, \ Q = x^2$	
		$\therefore$ Integrating Factor $= e^{\int Pdx}$	
		$\qquad = e^{\int \frac{1}{x}dx}$	
		$\qquad = e^{\log x}$	
		$\qquad = x$	1
		$\therefore$ Solution is,	
		$y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dx + c$	
		$\therefore y \cdot x = \int x^2 \cdot x dx + c$	1
		$\therefore xy = \int x^3 dx + c$	
		$\therefore xy = \dfrac{x^4}{4} + c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	The velocity of a particle is given by $v = t^2 - 6t + 7$. Find distance covered in 3 seconds.	**06**
	Ans	$v = t^2 - 6t + 7$	
		$\therefore v = \dfrac{dx}{dt} = t^2 - 6t + 7$	1
		$\therefore dx = \left(t^2 - 6t + 7\right)dt$	
		$\therefore \int dx = \int \left(t^2 - 6t + 7\right)dt$	1
		$\therefore x = \dfrac{t^3}{3} - 3t^2 + 7t + c$	1
		at $t = 0$, $x = 0$ $\therefore c = 0$	1
		$\therefore x = \dfrac{t^3}{3} - 3t^2 + 7t$	1
		at $t = 3$	
		$\therefore x = \dfrac{(3)^3}{3} - 3(3)^2 + 7(3)$	
		$\therefore x = 3$	1

6.		**Attempt any <u>TWO</u> of the following:**	**12**
	a) (i)	Express in polar form, $Z = 1 + i\sqrt{3}$	**03**
	Ans	$Z = 1 + i\sqrt{3}$	
		Comparing with $Z = x + iy$	
		$\therefore x = 1, y = \sqrt{3}$	
		$r = \sqrt{x^2 + y^2}$	
		$r = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$	1
		$\theta = \tan^{-1}\left\|\dfrac{y}{x}\right\|$	
		$\theta = \tan^{-1}\left\|\dfrac{\sqrt{3}}{1}\right\|$	
		$\theta = \tan^{-1}\sqrt{3}$	
		$\theta = 60^0$ or $\dfrac{\pi}{3}$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a) (i)	In polar form, $Z = r\left(\cos\theta + i\sin\theta\right)$ $Z = 2\left(\cos 60^0 + i\sin 60^0\right)$ or $Z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$	1
	a)(ii) Ans	Find $L\left\{\sin 3t + \cos 2t\right\}$ $L\left\{\sin 3t + \cos 2t\right\}$ $= \dfrac{3}{s^2 + 3^2} + \dfrac{s}{s^2 + 2^2}$ $= \dfrac{3}{s^2 + 9} + \dfrac{s}{s^2 + 4}$	**03** 2 1
	b) Ans	Find $L^{-1}\left\{\dfrac{2s+3}{(s+2)(s+6)}\right\}$ $L^{-1}\left\{\dfrac{2s+3}{(s+2)(s+6)}\right\}$ Let $\dfrac{2s+3}{(s+2)(s+6)} = \dfrac{A}{s+2} + \dfrac{B}{s+6}$ $\therefore 2s+3 = (s+6)A + (s+2)B$ Put $s = -2$ $\therefore -1 = 4A$ $\therefore A = -\dfrac{1}{4}$ Put $s = -6$ $-9 = -4B$ $\therefore B = \dfrac{9}{4}$ $\therefore \dfrac{2s+3}{(s+2)(s+6)} = \dfrac{\dfrac{-1}{4}}{s+2} + \dfrac{\dfrac{9}{4}}{s+6}$	**06** ½ 1 1 ½

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Subject Name: Applied Mathematics   **Model Answer**   **Subject Code:** 22210

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore L^{-1}\left\{\dfrac{2s+3}{(s+2)(s+6)}\right\}=-\dfrac{1}{4}L^{-1}\left\{\dfrac{1}{s+2}\right\}+\dfrac{9}{4}L^{-1}\left\{\dfrac{1}{s+6}\right\}$	1
		$=-\dfrac{1}{4}e^{-2t}+\dfrac{9}{4}e^{-6t}$	2
		--	
	c)	Solve the differential equation using Laplace Transformation.	**06**
		$\dfrac{dy}{dt}-3y=t\cdot e^{-2t}$, $y(0)=0$	
	Ans	$\dfrac{dy}{dt}-3y=t\cdot e^{-2t}$	
		$\therefore L\left\{\dfrac{dy}{dt}-3y\right\}=L\left\{t\cdot e^{-2t}\right\}$	
		$\therefore sL(y)-y(0)-3L(y)=(-1)^{1}\dfrac{d}{ds}\left(\dfrac{1}{s+2}\right)$	1
		$\therefore sL(y)-0-3L(y)=-1\cdot\dfrac{-1}{(s+2)^{2}}$	½
		$\therefore (s-3)L(y)=\dfrac{1}{(s+2)^{2}}$	
		$\therefore L(y)=\dfrac{1}{(s+2)^{2}(s-3)}$	1
		$\therefore y=L^{-1}\left\{\dfrac{1}{(s+2)^{2}(s-3)}\right\}$	
		Let $\dfrac{1}{(s+2)^{2}(s-3)}=\dfrac{A}{s+2}+\dfrac{B}{(s+2)^{2}}+\dfrac{C}{s-3}$	
		$\therefore 1=A(s+2)(s-3)+B(s-3)+C(s+2)^{2}$	
		Put $s=-2$	
		$\therefore 1=B(-2-3)$	
		$\therefore B=\dfrac{-1}{5}$	½
		Put $s=3$	
		$\therefore 1=C(3+2)^{2}$	

WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics	Model Answer	Subject Code:	22210

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore C = \dfrac{1}{25}$ Put $s = 0$ $\therefore 1 = A(-6) + B(-3) + C(4)$ $\therefore 1 = -6A - \dfrac{1}{5}(-3) + \dfrac{1}{25}(4)$ $\therefore 1 - \dfrac{3}{5} - \dfrac{4}{25} = -6A$ $\therefore \dfrac{6}{25} = -6A$ $\therefore A = \dfrac{-1}{25}$ $\therefore \dfrac{1}{(s+2)^2(s-3)} = \dfrac{\frac{-1}{25}}{s+2} + \dfrac{\frac{-1}{5}}{(s+2)^2} + \dfrac{\frac{1}{25}}{s-3}$ $L^{-1}\left\{\dfrac{1}{(s+2)^2(s-3)}\right\} = L^{-1}\left\{\dfrac{\frac{-1}{25}}{s+2} + \dfrac{\frac{-1}{5}}{(s+2)^2} + \dfrac{\frac{1}{25}}{s-3}\right\}$ $= \dfrac{-1}{25}e^{-2t} - \dfrac{1}{5}t \cdot e^{-2t} + \dfrac{1}{25}e^{3t}$	½ ½ 2

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.