WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	If $f(x)=16^x-\log_2 x$ find $f\left(\frac{1}{4}\right)$	**02**
	Ans	$f(x)=16^x-\log_2 x$ $f\left(\frac{1}{4}\right)=16^{\frac{1}{4}}-\log_2\left(\frac{1}{4}\right)$ $=2$	 1 1
		--	
	b)	If $f(x)=ax^2-bx-1, f(2)=5, f(-2)=10$ find a and b.	**02**
	Ans	$f(x)=ax^2-bx-1$ $f(2)=5$ $4a-2b-1=5$ $4a-2b=6$ -----------------(1) $f(-2)=10$ $4a+2b-1=10$ $4a+2b=11$ ----------------(2) From (1) and (2) $a=\frac{17}{8}$ $b=\frac{5}{4}$	 ½ ½ ½ ½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	Find $\frac{dy}{dx}$, if $y = x\sin^{-1}x$	**02**
	Ans	$y = x\sin^{-1}x$	
		$\frac{dy}{dx} = x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.1$	2
		$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x$	
		--	
	c)	Evaluate : $\int \frac{dx}{3x^2+4}$	**02**
	Ans	$\int \frac{dx}{3x^2+4}$	
		$= \int \frac{dx}{\left(\sqrt{3}x\right)^2 + 2^2}$	½
		$= \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)\frac{1}{\sqrt{3}} + c$	1
		$= \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$	½
		--	
	e)	Evaluate $\int \sin^3 x\, dx$	**02**
	Ans	$\int \sin^3 x\, dx$	
		since $\sin 3x = 3\sin x - 4\sin^3 x$ $\quad \therefore \sin^3 x = \frac{1}{4}\left(3\sin x - \sin 3x\right)$	
		$\therefore \int \frac{1}{4}\left(3\sin x - \sin 3x\right) dx$	1
		$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$	1
		OR	
		$\int \sin^3 x\, dx$	
		$= \int \sin^2 x\ \sin x\, dx$	
		$= \int \left(1 - \cos^2 x\right)\sin x\, dx$	½
		Put $\cos x = t$	
		$\therefore -\sin x dx = dt$	
		$\therefore \sin x dx = -dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	$\therefore \int\left(1-t^2\right)\left(-dt\right)$	
		$=-\int\left(1-t^2\right)dt$	
		$=-\left(t-\frac{t^3}{3}\right)+c$	½
		$=-\left(\cos x-\frac{\cos^3 x}{3}\right)+c$	½
		--	
	f)	Find the volume obtained by revolving the area under the curve $9x^2-4y^2=36$ in the interval from $x=2$ to , $x=4$ about $x-$axis	**02**
	Ans	$9x^2-4y^2=36$	
		$y^2=\frac{9}{4}\left(x^2-4\right)$	
		$\text{volume}=\pi\int_a^b y^2dx$	
		$=\pi\int_2^4\frac{9}{4}\left(x^2-4\right)dx$	
		$=\frac{9\pi}{4}\left[\frac{x^3}{3}-4x\right]_2^4$	½
		$=\frac{9\pi}{4}\left[\left(\frac{4^3}{3}-4(4)\right)-\left(\frac{2^3}{3}-4(2)\right)\right]$	1
		$=24\pi$	½
		--	
	g)	Find order and degree of the differential equation $\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{\frac{3}{2}}$	**02**
	Ans	$\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{\frac{3}{2}}$	
		Squaring on both sides	
		$\left(\frac{d^2y}{dx^2}\right)^2=\left(y+\frac{dy}{dx}\right)^3$	
		$\therefore Order=2$	1
		$\therefore Degree=2$	1
		--	
2.		**Attempt any THREE of the following:**	**12**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	If $x^{p}y^{q}=(x+y)^{p+q}$ show that $\frac{dy}{dx}=\frac{y}{x}$	**04**
	Ans	$x^{p}y^{q}=(x+y)^{p+q}$	
		$\log\left(x^{p}y^{q}\right)=\log(x+y)^{p+q}$	
		$\log x^{p}+\log y^{q}=(p+q)\log(x+y)$	½
		$p\log x+q\log y=(p+q)\log(x+y)$	½
		$p\frac{1}{x}+q\frac{1}{y}\frac{dy}{dx}=(p+q)\left(\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\right)$	1
		$\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}+\frac{p+q}{x+y}\frac{dy}{dx}$	
		$\frac{q}{y}\frac{dy}{dx}-\frac{p+q}{x+y}\frac{dy}{dx}=\frac{p+q}{x+y}-\frac{p}{x}$	½
		$\frac{dy}{dx}\left(\frac{q}{y}-\frac{p+q}{x+y}\right)=\frac{p+q}{x+y}-\frac{p}{x}$	
		$\frac{dy}{dx}\left(\frac{qx+qy-py-qy}{y(x+y)}\right)=\frac{px+qx-px-py}{x(x+y)}$	½
		$\frac{dy}{dx}\left(\frac{qx-py}{y}\right)=\frac{qx-py}{x}$	
		$\frac{dy}{dx}=\frac{y}{x}$	1
		---	
	b)	If $y=3\sin\theta-2\sin^{3}\theta,\ x=3\cos\theta-2\cos^{3}\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$y=3\sin\theta-2\sin^{3}\theta$	
		$\therefore\frac{dy}{d\theta}=3\cos\theta-6\sin^{2}\theta\cdot\cos\theta$	1
		$=3\cos\theta\left(1-2\sin^{2}\theta\right)$	½
		$x=3\cos\theta-2\cos^{3}\theta$	
		$\frac{dx}{d\theta}=-3\sin\theta+6\cos^{2}\theta\cdot\sin\theta$	1
		$=-3\sin\theta\left(1-2\cos^{2}\theta\right)$	½
		$\therefore\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{3\cos\theta\left(1-2\sin^{2}\theta\right)}{-3\sin\theta\left(1-2\cos^{2}\theta\right)}$	
		$=\frac{3\cos\theta(\cos2\theta)}{-3\sin\theta(-\cos2\theta)}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	$=\cot\theta$	
		$\therefore at\ \theta=\frac{\pi}{4}$	
		$\therefore \frac{dy}{dx}=\cot\frac{\pi}{4}$	
		$=1$	½
		---	
	c)	Find the radius of curvature of the curve $xy=c$ at point $\left(c,c\right)$	**04**
	Ans	$xy=c$	
		$x\frac{dy}{dx}+y.1=0$	
		$\frac{dy}{dx}=-\frac{y}{x}$	1
		$\frac{d^2y}{dx^2}=-\frac{\left[x\frac{dy}{dx}-y\right]}{x^2}$	1
		at point $\left(c,c\right)$	
		$\frac{dy}{dx}=-\frac{c}{c}=-1$	½
		$\frac{d^2y}{dx^2}=-\frac{\left[c\left(-1\right)-c\right]}{c^2}$	
		$=\frac{2}{c}$	½
		$\therefore$ radius of curvature $=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$=\frac{\left(1+\left(-1\right)^2\right)^{\frac{3}{2}}}{\frac{2}{c}}$	
		$=2^{\frac{1}{2}}c$ or $\sqrt{2}c$	1
		---	
	d)	Discuss the maxima and minima of the function "$\tan x-2x$"	**04**
	Ans	Let $y=\tan x-2x$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\therefore \frac{dy}{dx} = \sec^2 x - 2$	½
		$\therefore \frac{d^2y}{dx^2} = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$	½
		consider $\frac{dy}{dx} = 0$	
		$\therefore \sec^2 x - 2 = 0$	
		$\therefore \sec^2 x = 2$	
		$\therefore \sec x = \sqrt{2}, -\sqrt{2}$	1
		$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$	
		at $x = \frac{\pi}{4}$	
		$\therefore \frac{d^2y}{dx^2} = 2\sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} = 2(2)(1) = 4 > 0$	½
		$\therefore$ function is minimum at $x = \frac{\pi}{4}$	
		$\therefore y_{\min} = \tan \frac{\pi}{4} - 2\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}$	½
		at $x = \frac{3\pi}{4}$	
		$\therefore \frac{d^2y}{dx^2} = 2\sec^2\left(\frac{3\pi}{4}\right)\tan\left(\frac{3\pi}{4}\right) = 2(2)(-1) = -4 < 0$	½
		$\therefore$ function is maximum at $x = \frac{3\pi}{4}$	
		$\therefore y_{\max} = \tan\left(\frac{3\pi}{4}\right) - 2\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$	½
		--	
3.		**Attempt any THREE of the following:**	**12**
	a)	Find the equation of tangent and normal to the curve $y = x(2-x)$ at point $(2,0)$	**04**
	Ans	$y = x(2-x)$	
		$\therefore y = 2x - x^2$	
		$\therefore \frac{dy}{dx} = 2 - 2x$	1
		at $(2,0)$	
		$\therefore \frac{dy}{dx} = 2 - 2(2)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$\therefore \frac{dy}{dx}=-2$	½
		$\therefore$ slope of tangent , $m=-2$	
		Equation of tangent at $(2,0)$ is	
		$y-0=-2(x-2)$	
		$\therefore y=-2x+4$	
		$\therefore 2x+y-4=0$	1
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{1}{2}$	½
		Equation of normal at $(2,0)$ is	
		$y-0=\frac{1}{2}(x-2)$	
		$\therefore 2y=x-2$	
		$\therefore x-2y-2=0$	1
	b)	Find $\frac{dy}{dx}$, $y=\left(\sin^{-1}x\right)^{x}+\left(\cos x\right)^{\sin x}$	**04**
	Ans	Let $u=\left(\sin^{-1}x\right)^{x}$	
		$\therefore \log u=\log\left(\sin^{-1}x\right)^{x}$	
		$\therefore \log u=x\log\left(\sin^{-1}x\right)$	½
		$\therefore \frac{1}{u}\frac{du}{dx}=x\cdot\frac{1}{\sin^{-1}x}\frac{1}{\sqrt{1-x^{2}}}+\log\left(\sin^{-1}x\right).1$	1
		$\therefore \frac{1}{u}\frac{du}{dx}=\frac{x}{\sqrt{1-x^{2}}\sin^{-1}x}+\log\left(\sin^{-1}x\right)$	
		$\therefore \frac{du}{dx}=u\left(\frac{x}{\sqrt{1-x^{2}}\sin^{-1}x}+\log\left(\sin^{-1}x\right)\right)$	
		$\therefore \frac{du}{dx}=\left(\sin^{-1}x\right)^{x}\left(\frac{x}{\sqrt{1-x^{2}}\sin^{-1}x}+\log\left(\sin^{-1}x\right)\right)$	½
		Let $v=\left(\cos x\right)^{\sin x}$	
		$\therefore \log v=\log\left(\cos x\right)^{\sin x}$	
		$\therefore \log v=\sin x\log\left(\cos x\right)$	½
		$\therefore \frac{1}{v}\frac{dv}{dx}=\sin x\cdot\frac{1}{\cos x}.\left(-\sin x\right)+\log\left(\cos x\right)\cdot\cos x$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\therefore \frac{1}{v}\frac{dv}{dx} = -\sin x\tan x + \log\left(\cos x\right)\cdot\cos x$	
		$\therefore \frac{dv}{dx} = v\left[-\sin x\tan x + \log\left(\cos x\right)\cdot\cos x\right]$	
		$\therefore \frac{dv}{dx} = \left(\cos x\right)^{\sin x}\left[-\sin x\tan x + \log\left(\cos x\right)\cdot\cos x\right]$	½
		$\therefore \frac{dy}{dx} = \left(\sin^{-1}x\right)^{x}\left(\frac{x}{\sqrt{1-x^2}\,\sin^{-1}x} + \log\left(\sin^{-1}x\right)\right)$ $+\left(\cos x\right)^{\sin x}\left[-\sin x\tan x + \log\left(\cos x\right)\cdot\cos x\right]$	½
		--	
	c)	If $y = \tan^{-1}\left[\frac{5x-4}{5+4x}\right]$ find $\frac{dy}{dx}$	**04**
	Ans	$y = \tan^{-1}\left[\frac{5x-4}{5+4x}\right]$	
		$y = \tan^{-1}\left[\frac{x-\frac{4}{5}}{1+\frac{4}{5}x}\right]$	1
		$\text{y} = \tan^{-1}x - \tan^{-1}\frac{4}{5}$	2
		$\frac{dy}{dx} = \frac{1}{1+x^2} - 0$	1
		$\frac{dy}{dx} = \frac{1}{1+x^2}$	
		--	
	d)	Evaluate $\int \frac{\sec^2 x}{\left(1+\tan x\right)\left(2+\tan x\right)}dx$	**04**
	Ans	$\int \frac{\sec^2 x}{\left(1+\tan x\right)\left(2+\tan x\right)}dx$	
		Let $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	½
		$= \int \frac{1}{\left(1+t\right)\left(2+t\right)}dt$	
		Consider	
		$\frac{1}{\left(1+t\right)\left(2+t\right)} = \frac{A}{1+t} + \frac{B}{2+t}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
	d)	$\therefore 1 = A(2+t)+B(1+t)$	
		Put $t=-1$ $\therefore A=1$	1
		Put $t=-2$ $\therefore B=-1$	1
		$\therefore \frac{1}{(1+t)(2+t)} = \frac{1}{1+t}+\frac{-1}{2+t}$	
		$\therefore \int \frac{1}{(1+t)(2+t)}dt = \int\left(\frac{1}{1+t}+\frac{-1}{2+t}\right)dt$	
		$= 1\log(1+t)-1\log(2+t)+c$	1
		$= \log\left(\frac{1+t}{2+t}\right)+c$	
		$= \log\left(\frac{1+\tan x}{2+\tan x}\right)+c$	½
		OR	
		$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\ dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 xdx = dt$	½
		$\int \frac{1}{(1+t)(2+t)}dt$	
		$= \int \frac{1}{t^2+3t+2}\ dt$	½
		Third Term $= \frac{3^2}{4} = \frac{9}{4}$	
		$= \int \frac{1}{t^2+4t+\frac{9}{4}-\frac{9}{4}+2}\ dt$	½
		$= \int \frac{1}{\left(t+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}\ dt$	½
		$= \frac{1}{2.\frac{1}{2}}\log\left\|\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right\|+c$	1
		$= \log\left\|\frac{t+1}{t+2}\right\|+c$	
		$= \log\left\|\frac{\tan x+1}{\tan x+2}\right\|+c$	1
		--	
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int \frac{1}{2x^2+3x+1}dx$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)Ans	$\int\frac{1}{2x^2+3x+1}\,dx=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{1}{2}}\,dx$	
		Third term $=\left(\frac{1}{2}\times\frac{3}{2}\right)^2=\frac{9}{16}$	
		$=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{1}{2}}dx$	1
		$=\frac{1}{2}\int\frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$	1
		$=\frac{1}{2}\left[\frac{1}{2\left(\frac{1}{4}\right)}\log\left(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}\right)\right]+c$	1 1
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	
		OR	
		$\int\frac{1}{2x^2+3x+1}dx=\int\frac{1}{(2x+1)(x+1)}dx$	½
		Let $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$	
		$1=A(x+1)+B(2x+1)$	
		Put $x=\frac{-1}{2}$	1
		$\therefore A=2$	1
		Put $x=-1$	
		$\therefore B=-1$	
		$\frac{1}{(2x+1)(x+1)}=\frac{2}{2x+1}+\frac{-1}{x+1}$	½
		$\int\frac{1}{(2x+1)(x+1)}dx=\int\left(\frac{2}{2x+1}+\frac{-1}{x+1}\right)dx$	1
		$=\frac{2\log(2x+1)}{2}-\log(x+1)+c$	
		$=\log(2x+1)-\log(x+1)+c$	
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$\int \frac{1}{2x^2+3x+1}dx$	
	Ans	Third term $=\frac{(M.T.)^2}{4(F.T.)}=\frac{9}{8}$	
		$=\int \frac{1}{2x^2+3x+\frac{9}{8}-\frac{9}{8}+1}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\frac{1}{8}}dx$	
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\left(\frac{1}{\sqrt{8}}\right)^2}dx$	1
		$=\frac{1}{\sqrt{2}}\left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)}\log\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{8}}-\frac{1}{\sqrt{8}}}{\sqrt{2}x+\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{8}}}\right)\right]+c$	1
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	1
		---	
	b)	Evaluate $\int \frac{dx}{1+\sin x+\cos x}$	**04**
	Ans	$\int \frac{dx}{1+\sin x+\cos x}$	
		Put $\tan\frac{x}{2}=t \quad \therefore \sin x=\frac{2t}{1+t^2}\ ,\cos x=\frac{1-t^2}{1+t^2},\quad dx=\frac{2dt}{1+t^2}$	
		$\therefore \int \frac{dx}{1+\sin x+\cos x}=\int \frac{1}{1+\frac{2t}{1+t^2}+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	1
		$=\int \frac{2\,dt}{1+t^2+2t+1-t^2}\,dt$	
		$=2\int \frac{1}{2t+2}\,dt$	
		$=\int \frac{dt}{t+1}$	1
		$=\log(t+1)+c$	1
		$=\log\left(\tan\frac{x}{2}+1\right)+c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	Evaluate : $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$	**04**
	Ans	$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$	
		put $\sin^{-1}x=t \quad \therefore x=\sin t$	
		$\frac{1}{\sqrt{1-x^2}}dx=dt$	1
		$\int t\sin t\ dt$	
		$=t\int\sin tdt-\int\Big(\Big(\int\sin tdt\Big)\frac{d}{dt}(t)dt$	1
		$=t(-\cos t)-\int-\cos t\cdot 1\cdot dt$	½
		$=-t\cos t+\sin t+c$	½
		$=-\sin^{-1}x\cos\left(\sin^{-1}x\right)+\sin\left(\sin^{-1}x\right)+c$	1
		--	
	d)	Evaluate: $\int_0^{\frac{\pi}{2}}\frac{\tan x}{1+\tan x}dx$	**04**
	Ans	Let $I=\int_0^{\frac{\pi}{2}}\frac{\tan x}{1+\tan x}dx$	
		$=\int_0^{\frac{\pi}{2}}\frac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}dx$	½
		$\therefore I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}dx$ ---------------------(1)	
		$I=\int_0^{\pi/2}\frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}dx$	1
		$\therefore I=\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}dx$ ---------------------(2)	½
		add (1) and (2)	
		$I+I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}dx+\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$2I=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sin x+\cos x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	½
		$2I=\left[x\right]_0^{\pi/2}$	½
		$2I=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		--	
	e)	Evaluate: $\int\frac{x}{\left(x^2+4\right)\left(x^2+9\right)}dx$	**04**
	Ans	$\int\frac{x}{\left(x^2+4\right)\left(x^2+9\right)}dx$	
		Put $x^2=t$	
		$2xdx=dt$	
		$xdx=\frac{dt}{2}$	½
		$\int\frac{\frac{dt}{2}}{(t+4)(t+9)}$	
		$=\frac{1}{2}\int\frac{dt}{(t+4)(t+9)}$	
		$\frac{1}{(t+4)(t+9)}=\frac{A}{t+4}+\frac{B}{t+9}$	
		$1=A(t+9)+B(t+4)$	
		Put $t=-4\quad\therefore A=\frac{1}{5}$	1
		Put $t=-9\quad\therefore B=-\frac{1}{5}$	1
		$\frac{1}{(t+4)(t+9)}=\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}$	
		$\int\frac{dt}{(t+4)(t+9)}=\int\left(\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}\right)dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
	e)		
		$=\frac{1}{5}\log\left(t+4\right)-\frac{1}{5}\log\left(t+9\right)+c$	½
		$=\frac{1}{5}\log\left(x^2+4\right)-\frac{1}{5}\log\left(x^2+9\right)+c$	½
		--	
5.		**Attempt any TWO of the following:**	
		Find area of between the curve $y^2-2x=0$ and $y^2+4x-12=0$	**12**
	a)	$y^2=2x$ ---------(1)	**06**
	Ans	$y^2=12-4x$	
		$\therefore 2x=12-4x$	
		$\therefore 6x=12$	
		$\therefore x=2\ ,\ y=\pm2$	1
		$\therefore x=\frac{y^2}{2}\quad ,\quad x=\frac{12-y^2}{4}$	
		Area $A=\int_a^b\left(x_1-x_2\right)dy$	
		$\therefore A=\int_{-2}^{2}\left(\frac{12-y^2}{4}-\frac{y^2}{2}\right)dy$	1
		$\therefore A=\frac{3}{4}\int_{-2}^{2}\left(4-y^2\right)dy$	1
		$\therefore A=\frac{3}{4}\left(4y-\frac{y^3}{3}\right)_{-2}^{2}$	1
		$\therefore A=\frac{3}{4}\left(4\left(2\right)-\frac{\left(2\right)^3}{3}-\left(4\left(-2\right)-\frac{\left(-2\right)^3}{3}\right)\right)$	1
		$\therefore A=8$	1
		--	
	b)	Attempt the following:	**06**
	(i)	Form the differential equation If $y=A\cos\left(\log x\right)+B\sin\left(\log x\right)$	**03**
	Ans	$y=A\cos\left(\log x\right)+B\sin\left(\log x\right)$	
		$\therefore\ \frac{dy}{dx}=-\frac{A\sin\left(\log x\right)}{x}+\frac{B\cos\left(\log x\right)}{x}$	1
		$\therefore x\frac{dy}{dx}=-A\sin\left(\log x\right)+B\cos\left(\log x\right)$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{A\cos\left(\log x\right)}{x}-\frac{B\sin\left(\log x\right)}{x}$	1
		$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-A\cos\left(\log x\right)-B\sin\left(\log x\right)$	

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code:** 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)i)	$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-y$ $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ --	½
	b)ii)	Solve $x\log x\frac{dy}{dx}+y=2\log x$	**03**
	Ans	$x\log x\frac{dy}{dx}+y=2\log x$ $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x}$ $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$	½
		$I.F=e^{\int\frac{1}{x\log x}dx}$	½
		$=e^{\log(\log x)}=\log x$ Solution is,	½
		$y.\log x=\int\frac{2}{x}.\log xdx$ Let $\mathrm{I}_1=\int\frac{2}{x}.\log xdx$ Put $\log x=t$	½
		$\frac{1}{x}dx=dt$ $\therefore \mathrm{I}_1=2\int tdt$ $=2\frac{t^2}{2}+c$	½
		$=(\log x)^2+c$ $y.\log x=(\log x)^2+c$ --	½
	c)	A circular column of radius 'x' and having depth y support a load.The equation of equilibrium is $2\frac{dy}{dx}-kx=0$ where 'k' is constant.Find the relation between x and y.	**06**
	Ans	$2\frac{dy}{dx}-kx=0$ $2\frac{dy}{dx}=kx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\frac{dy}{dx}=\frac{k}{2}x$	½
		$dy=\frac{k}{2}xdx$	1
		$\int dy=\int\frac{k}{2}xdx$	1
		$y=\frac{k}{2}\frac{x^2}{2}+c_1$	2
		$4y=kx^2+4c$	1
		$4y=kx^2+c$	½

6.		**Attempt any TWO of the following:**	**12**
	a)	Using Simpson's $1/3^{rd}$ rule evaluate $\int_0^2\frac{1}{1+x^3}dx$ with $n=4$.	**06**
	Ans	Let $y=\frac{1}{1+x^3}$ $a=0,b=2$ and $n=4$	
		$\therefore h=\frac{b-a}{n}=\frac{2-0}{4}=\frac{1}{2}$	1
		$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 0 & \frac{1}{2} & 1 & \frac{3}{2} & 2 \\ \hline y=\frac{1}{1+x^3} & 1 & \frac{8}{9} & \frac{1}{2} & \frac{8}{35} & \frac{1}{9} \\ \hline\end{array}$	2
		Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		$\therefore\int_0^1 f(x)dx=\frac{1/2}{3}\left[\left(1+\frac{1}{9}\right)+4\left(\frac{8}{9}+\frac{8}{35}\right)+2\left(\frac{1}{2}\right)\right]$	2
		$\therefore\int_0^1\frac{1}{1+x^3}dx=1.0968$	1
		OR	
		Let $y=\frac{1}{1+x^3}$ $a=0,b=2$ and $n=4$	
		$\therefore h=\frac{b-a}{n}=\frac{2-0}{4}=\frac{1}{2}=0.5$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme

6. a)

x	0	0.5	1	1.5	2
$y=\frac{1}{1+x^3}$	1	0.8889	0.5	0.2286	0.1111

2

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore\int_0^1 f(x)dx=\frac{0.5}{3}\left[(1+0.1111)+4(0.8889+0.2286)+2(0.5)\right]$$ 2

$$\int_0^1\frac{1}{1+x^2}dx=1.0969$$ 1

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b) Using Simpson's $3/8^{th}$ rule,evaluate $\int_0^{\frac{\pi}{2}}\cos x dx$ with $n=8$ **06**

Ans

Here $n=8$

$y=\cos x \quad a=0, \quad b=\frac{\pi}{2}$

$$\therefore h=\frac{b-a}{n}=\frac{\frac{\pi}{2}-0}{8}=\frac{\pi}{16}$$ 1

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$\frac{5\pi}{16}$	$\frac{3\pi}{8}$	$\frac{7\pi}{16}$	$\frac{\pi}{2}$
$y=\cos x$	1	0.9808	0.9239	0.8315	0.7071	0.5556	0.3827	0.1951	0

2

Using Simpson's $3/8^{th}$ rule.

$$\int_a^b f(x)dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$$

$$\therefore\int_0^{\frac{\pi}{2}}\cos x dx=\frac{3\left(\frac{\pi}{16}\right)}{8}\begin{bmatrix}(1+0)+3(0.9808+0.9239+0.7071+0.5556+0.1951)\\+2(0.8315+0.3827)\end{bmatrix}$$ 2

$$\therefore\int_0^{\frac{\pi}{2}}\cos x dx=0.9952$$ 1

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c) Attempt the following: **06**

Q. No.	Sub Q. N.	Answer	Marking Scheme

6. c)(i) Using Trapezoidal rule, evaluate $\int_{-1}^{1}\left(1+x+x^2+x^3\right)\,dx,$ by taking $n=2.$ — 03

Ans

$y=1+x+x^2+x^3 \quad a=-1, \quad b=1$

$\therefore h=\frac{b-a}{n}=\frac{1+1}{2}=1$ — ½

x	-1	0	1
$y=1+x+x^2+x^3$	0	1	4

— 1

$$\int_a^b f(x)\,dx=\frac{h}{2}\left[\left(y_0+y_n\right)+2\left(y_1+y_2+...+y_{n-1}\right)\right]$$

$a=-1, b=1$ and $h=1$

$$\therefore \int_{-1}^{1}\left(1+x+x^2+x^3\right)dx=\frac{1}{2}\left[\left(0+4\right)+2\left(1\right)\right]$$ — ½

$$=3$$ — 1

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ii) Using Simpson's $1/3^{rd}$ rule evaluate $\int_1^3 \frac{dx}{x}$ taking $h=0.5.$

Ans

Let $y=\frac{1}{x}$, $\mathrm{h}=0.5, a=1, b=3$

x	1	1.5	2	2.5	3
$y=\frac{1}{x}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

— 1

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)\,dx=\frac{h}{3}\left[\left(y_0+y_n\right)+4\left(y_1+y_3+...+y_{n-1}\right)+2\left(y_2+y_4+...+y_{n-2}\right)\right]$$

$$\therefore \int_1^3 \frac{dx}{x}=\frac{0.5}{3}\left[\left(1+\frac{1}{3}\right)+4\left(\frac{2}{3}+\frac{2}{5}\right)+2\left(\frac{1}{2}\right)\right]$$ — 1

$$\int_1^3 \frac{dx}{x}=1.1$$ — 1

OR

Let $y=\frac{1}{x}$, $\mathrm{h}=0.5, a=1, b=3$

x	1	1.5	2	2.5	3
$y=\frac{1}{x}$	1	0.6667	0.5	0.4	0.3333

— 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)(ii)	Using Simpson's $1/3^{rd}$ rule $\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		$\therefore \int_1^3 \frac{dx}{x} = \frac{0.5}{3}\left[(1+0.3333)+4(0.6667+0.4)+2(0.5)\right]$	1
		$\int_1^3 \frac{dx}{x} = 1.1$	1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
