Model Answers

Winter – 2019 Examinations

Subject & Code: Electrical Circuits (22324)

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Model Answers
Winter – 2019 Examinations
Subject & Code: Electrical Circuits (22324)

1 Attempt any TEN of the following: 20

1 a) Define Conductance and Susceptance related to AC circuit and state their units.

Ans:-

Conductance (G):

It is defined as the real part of the admittance (Y). ½ Mark for definition

It is also defined as the ability of the purely resistive circuit to pass the alternating current. ½ Mark for unit

OR

It is also defined as the ratio of resistance to the square of the impedance.

In general, Conductance, G= $\frac{R}{Z^2}$ siemen. Its unit is **siemen (S).**

Susceptance (B):

It is imaginary part of the admittance (Y). ½ Mark for definition

It is defined as the ability of the purely reactive circuit (purely capacitive or purely inductive) to admit alternating current. ½ Mark for unit

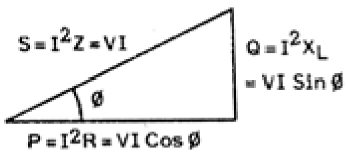
OR

It is also defined as the ratio of reactance to the square of the impedance.

In general, Susceptance (B) = $\frac{X}{Z^2}$ siemen. Its unit is **siemen (S).**

1 b) Draw power triangle for R-L series circuit. Write equation of power in rectangular form.

Ans: 1 Mark for power triangle



1 Mark for equation

$S = P + jQ$

$VI = VI\cos\phi + j\,VI\sin\phi$

$I^2Z = I^2R + j\ I^2X_L$

1 c) Express an instantaneous value of an alternating current varying sinusoidally in terms of its maximum value, frequency and time.

Ans:

$$i = I_m \sin(\omega t \pm \Phi) \text{ amp}$$

where, i = Instantaneous value 1 Mark for equation

I_m = Maximum value

ω = Angular frequency in rad/sec = 2πf

f = frequency in cycles/sec or Hz 1 Mark for terms

t = time in sec

Φ = phase angle

1 d) State relationship between line and phase values of voltage and current in balanced delta connection.

Ans:

Balanced Delta Connection:

$$\text{Line voltage} = \text{Phase Voltage}$$ 1 Mark

$$\text{i.e } V_L = V_{ph}$$

$$\text{Line current} = \sqrt{3}\ (\text{Phase current})$$ 1 Mark

$$\text{i.e } I_L = \sqrt{3} I_{ph}$$

1 e) Distinguish clearly between loop and mesh.

Ans:

Distinction between Loop & Mesh:

Sr. No.	Loop	Mesh
1	A loop is any closed path in a circuit, in which no node is encountered more than once	A mesh is a loop that has no other loops inside of it
2	Every loop is not a mesh	Every mesh is a loop
3	Loops are used in a more general way for circuit analysis	Meshes are used to analyze planar circuits

1 Mark for each of any two points = 2 Marks

1 f) State the value of internal resistance of (i) Ideal Voltage Source and (ii) Ideal Current Source.

Ans:

Value of Internal Resistance of Ideal Voltage Source $R_s = 0$

Value of Internal Resistance of Ideal Current Source $R_s = \infty$ 1 Mark each

1 g) State Norton's Theorem.

Ans:

Norton's Theorem:

Any two terminal circuit having number of linear impedances and sources (voltage, current, dependent, independent) can be represented by a simple equivalent circuit consisting of a single current source I_N in parallel with an impedance Z_N across the two terminals, where the source current I_N is equal to the short circuit current caused by internal sources when the two terminals are short circuited and the value of the parallel impedance Z_N is equal to the impedance of the circuit while looking back into the circuit across the two terminals, when the internal independent voltage sources are replaced by short-circuits and independent current sources by open circuits.

2 Marks for correct statement

2 Attempt any THREE of the following: 12

2 a) With neat diagram, explain the phasor representation of sinusoidal quantity.

Ans:

Phasor Representation of Sinusoidal Quantity:

Model Answers

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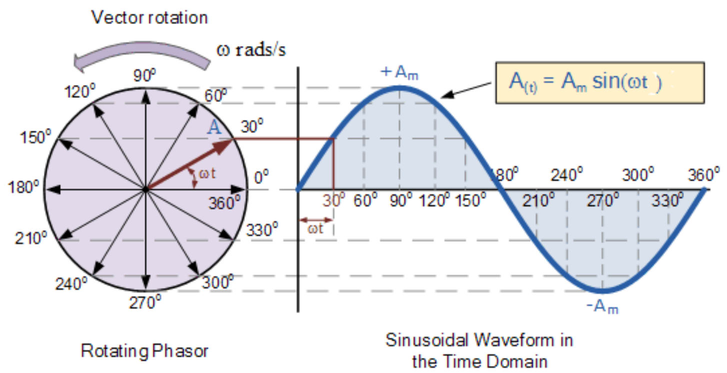


2 Marks for diagram

OR Equivalent Figure

When number of waveforms are drawn in the same figure, the complexity of diagram increases and it becomes very difficult to extract the information from the waveforms. Therefore, to extract the same information, simplified alternate approach is preferred, called "Phasor representation of Sinusoidal quantity".

2 Marks for explanation

A sinusoidal quantity is represented by a rotating vector or rotating phasor "A" whose length is equal to the amplitude of the quantity "A_m", as shown above. The points on the waveform are represented by the positions of the phasor during rotation drawn from the same reference point. The phasor making an angle of "ωt" with respect to positive x-axis reference, represents the instantaneous value of the quantity at an angle of "ωt" from its zero value, as shown above. In fact, the vertical component of the phasor represents the magnitude of the quantity at that particular instant. From the above diagram, it is clear that the vertical component of the phasor is "$A_m \sin(\omega t)$" which is the instantaneous value of the quantity at instant "ωt".

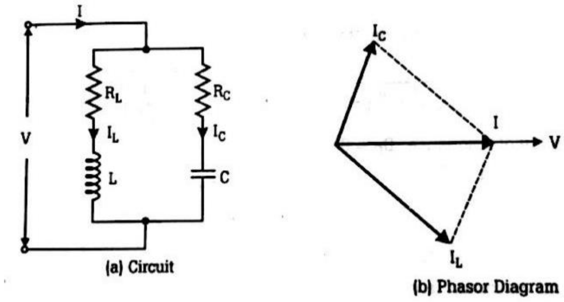
The speed of rotation of the phasor is equal to ω rad/sec where $\omega = 2\pi f$.

One rotation of the phasor corresponds to one cycle of the alternating waveform as shown in figure.

2 b) For a parallel circuit consisting of an inductive branch (RL) in parallel with a capacitive branch (RC), draw phasor diagram and derive equation for resonant frequency.

Ans:

Parallel Resonance in RL-RC parallel circuit:



Parallel Resonance for RL-RC Parallel Circuit

1 Mark for phasor diagram

The circuit diagram and phasor diagram is as shown in the figure. Under parallel resonance (anti-resonance) condition, the circuit will take an input current (I) in phase with the applied voltage (V). At resonance, the circuit impedance becomes purely resistive in spite of presence of L & C and the circuit power factor becomes unity.

The admittance of inductive branch is:

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L} = \frac{1}{R_L + j\omega L} = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2}$$

The admittance of capacitive branch is:

$$Y_L = \frac{1}{Z_C} = \frac{1}{R_C - jX_C} = \frac{1}{R_C - j\frac{1}{\omega C}} = \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

Total admittance of the parallel circuit:

$$Y = Y_L + Y_C = \left[\frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}\right]$$

$$= \left[\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right] - j\left[\frac{\omega L}{R_L^2 + \omega^2 L^2} - \frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}\right]$$

3 Marks for stepwise derivation

At resonance, $\omega = \omega_{ar}$ (anti-resonant angular frequency), the reactive term must be zero.

$$\therefore \left[\frac{\omega_{ar} L}{R_L^2 + \omega_{ar}^2 L^2} - \frac{\frac{1}{\omega_{ar} C}}{R_C^2 + \frac{1}{\omega_{ar}^2 C^2}}\right] = 0$$

$$\therefore \frac{\omega_{ar} L}{R_L^2 + \omega_{ar}^2 L^2} = \frac{\frac{1}{\omega_{ar} C}}{R_C^2 + \frac{1}{\omega_{ar}^2 C^2}}$$

$$\therefore \frac{\omega_{ar} L}{R_L^2 + \omega_{ar}^2 L^2} = \frac{\omega_{ar} C}{\omega_{ar}^2 C^2 R_C^2 + 1}$$

$$\therefore L\omega_{ar}^2 C^2 R_C^2 + L = CR_L^2 + \omega_{ar}^2 C L^2$$

$$\therefore \omega_{ar}^2 [LC^2 R_C^2 - CL^2] = CR_L^2 - L$$

$$\therefore \omega_{ar}^2 = \frac{CR_L^2 - L}{[LC^2 R_C^2 - CL^2]} = \frac{L - CR_L^2}{[CL^2 - LC^2 R_C^2]}$$

$$= \frac{1}{LC}\left[\frac{L - CR_L^2}{L - CR_C^2}\right]$$

$$\therefore \omega_{ar} = \sqrt{\frac{1}{LC}\left[\frac{L - CR_L^2}{L - CR_C^2}\right]} \text{ rad/sec}$$

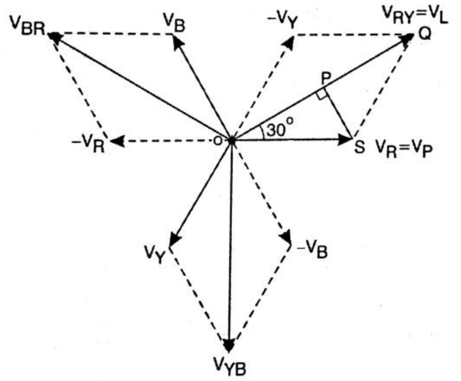
$\therefore$ Anti-resonant frequency $f_{ar} = \frac{\omega_{ar}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left[\frac{L - CR_L^2}{L - CR_C^2}\right]}\ Hz$

$$f_{ar} = \frac{\omega_{ar}}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left[\frac{L - CR_L^2}{L - CR_C^2}\right]}\ Hz$$

2 c) With the help of neat phasor diagram, derive the relationship between line and phase values of voltage in balanced star connection.

Ans:

Relationship Between Line voltage and Phase Voltage in Balanced Star Connection:



1 Mark for phasor diagram

Let V_R, V_Y and V_B be the phase voltages.

V_{RY}, V_{YB} and V_{BR} be the line voltages.

The line voltages are expressed as:

$V_{RY} = V_R - V_Y$

$V_{YB} = V_Y - V_B$

$V_{BR} = V_B - V_R$

In phasor diagram, the phase voltages are drawn first with equal amplitude and displaced from each other by 120°. Then line voltages are drawn as per the above equations. It is seen that the line voltage V_{RY} is the phasor sum of phase voltages V_R and $-V_Y$. We know that in parallelogram, the diagonals bisect each other with an angle of 90°.

3 Marks for stepwise derivation

Therefore in ΔOPS, ∠P = 90° and ∠O = 30°.

$$[OP] = [OS] \cos 30°$$

$$\text{Since } [OP] = V_L/2 \text{ and } [OS] = V_{ph}$$

$$\therefore \frac{V_L}{2} = V_{ph}\cos 30°$$

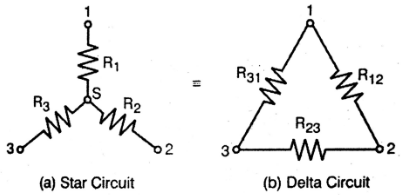
$$V_L = 2V_{ph}\frac{\sqrt{3}}{2}$$

$$\mathbf{V_L = \sqrt{3}\, V_{ph}}$$

Thus **Line voltage = $\sqrt{3}$ (Phase Voltage)**

2 d) State the equivalent delta connection for star connection of three resistances R_1, R_2 & R_3 with proper equations.

Ans:



(a) Star Circuit (b) Delta Circuit

1 mark for circuit diagram + 1 mark for each of 3 equations = 4 Marks

$$R_{12} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3}$$

$$R_{23} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1}$$

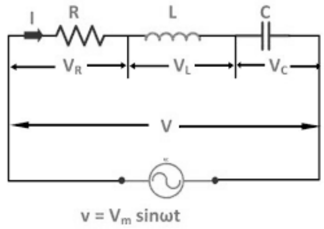
$$R_{31} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2}$$

3 Attempt any <u>THREE</u> of the following: 12

3 a) For series R-L-C circuit, draw neat circuit diagram. State the conditions for RLC series ckt. Draw phasor diagram and voltage triangle impedance triangle for any 1 condition.

Ans:

Circuit Diagram for R-L-C Series Circuit:



½ Mark for circuit diagram + ½ Mark for each of 3 circuit conditions = 2 Mark

Conditions for R-L-C Series Circuit:

(i) When $X_L > X_C$: Phase angle ∅ is positive and circuit will be inductive. In other words, in such a case, the circuit current I will lag behind the applied voltage V by angle ∅.

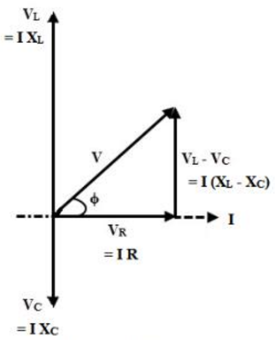
(ii) When $X_L < X_C$: Phase angle ∅ is negative and circuit will be capactive. In other words, in such a case, the circuit current I leads the applied voltage V by angle ∅.

(iii) When $X_L = X_C$: The circuit is purely resistive. In other word circuit current I and applied voltage V will be in phase i.e. $\emptyset = 0°$. The circuit will have unity power factor.

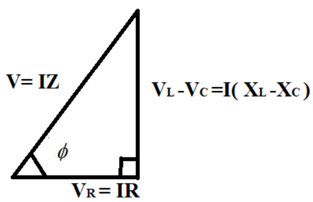
1 Mark for phasor diagram + ½ Mark each for voltage triangle & impedance triangle for any one condition = 2 Mark

Phasor Diagram, Voltage Triangle & Impedance Triangle:

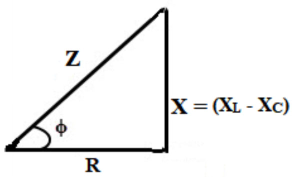
(i) Condition $X_L > X_C$



Vector Diagram

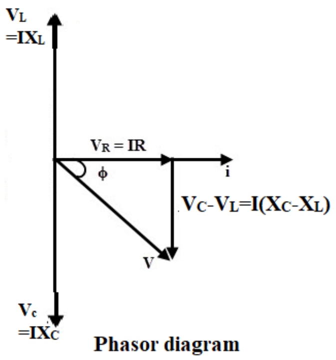


Voltage Triangle

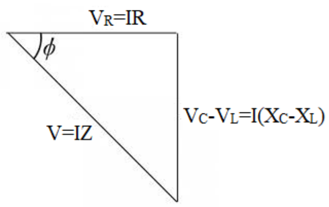


Impedance Triangle

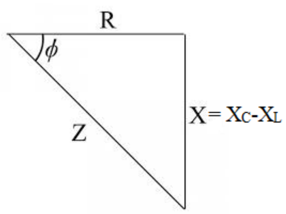
(ii) Condition $X_L < X_C$



Phasor diagram



Voltage Triangle



Impedance Triangle

3 b) State any four properties of Parallel Resonance.

Ans:

Properties of Parallel Resonance:

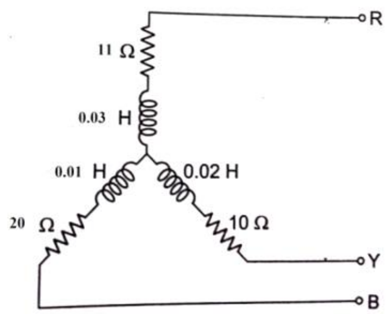
1. At resonance, the parallel RLC circuit behaves as purely resistive circuit.
2. At resonance, the Parallel RLC circuit power factor is unity.
3. At resonance, the parallel RLC circuit offers maximum total impedance Z=L/CR
4. At resonance, parallel RLC circuit draws minimum current from source, $I = \frac{V}{[L/_{CR}]}$
5. At resonance, in parallel RLC circuit, current magnification takes place.
6. The Q-factor for parallel resonant circuit is, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
7. Parallel RLC resonant circuit is Rejecter circuit.

1 Mark for each of any four properties = 4 Marks

3 c) With neat labeled diagram, explain unbalanced star connected load.

Ans:

Unbalanced Star connected Load:



1. When the magnitudes and phase angles of three impedances are differ from each other, then it is called as unbalanced load.
2. Phase angles of impedance are not equal.
3. For unbalanced load, the phase voltage is $\frac{1}{\sqrt{3}}$ of the line voltage.
4. All the voltages are fixed and line currents will not be equal nor will have a 120° phase difference.

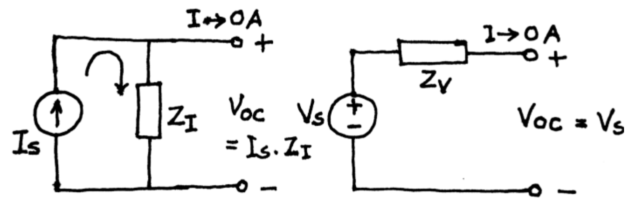
1 Mark for labeled circuit diagram + 3 Marks for explanation (any 3 points) = 4 Marks

3 d) With neat circuit diagram, explain how to convert a practical voltage source into an equivalent practical current source.

Ans:

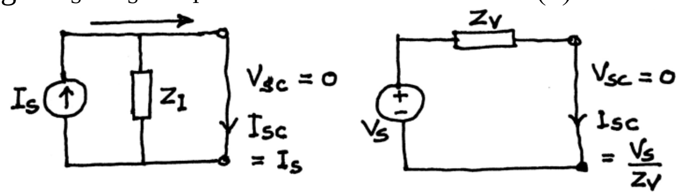
Conversion of practical voltage source into equivalent practical current source:

Let V_S be the practical voltage source magnitude and
Z_V be the internal series impedance of the voltage source.
I_S be the equivalent practical current source magnitude and
Z_I be the internal parallel impedance of current source.



1 Mark for each equation = 4 Marks

The open circuit terminal voltage of voltage source is $V_{OC} = V_S$
The open circuit terminal voltage of current source is $V_{OC} = I_S \times Z_I$
Therefore, we get $V_S = I_S \times Z_I$ …………………………(1)



The short circuit output current of voltage source is $I_{SC} = V_S / Z_V$
The short circuit output current of current source is $I_{SC} = I_S$
Therefore, we get $I_S = V_S / Z_V$ …………………………(2)
Therefore, we get $V_S = I_S \times Z_V$…………………………(3)
On comparing eq. (1) and (3), it is clear that $Z_I = Z_V = Z$ ……….. …..(4)

Thus the internal impedance of both the sources is same, and the magnitudes of the source voltage and current are related by Ohm's law, $V_S = I_S \times Z$

3 e) Explain the concept of "duality" in electric circuit with one example.

Ans:

Concept of duality:

When the two circuit elements are represented by mathematical equations of similar nature, then these elements are called dual elements of each other. 1 Mark

Examples:

(i) A resistance is represented by mathematical equation based on Ohm's law as, R = V/I and the conductance is represented by G = I/V. 1 Mark

(ii) A voltage across an inductance is represented by $v = L\frac{di}{dt}$ and the current through a capacitor is represented by $i = C\frac{dv}{dt}$ 1 Mark

On comparing the above equations we can form pairs of dual elements or quantities: 1 Mark

Resistance R ⟷ Conductance G
Inductance L ⟷ Capacitance C
Voltage v ⟷ Current i

OR

OR

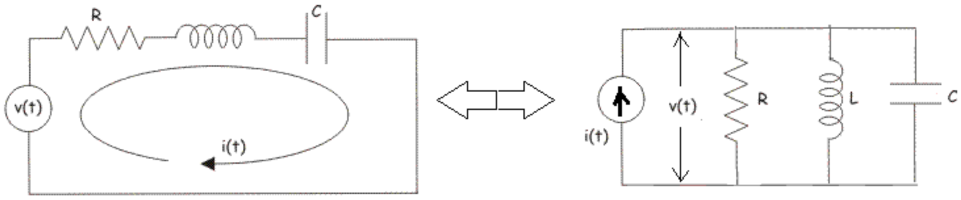
Similarly, we can apply this concept to electric circuits and say that when the two circuits are represented by similar mathematical equations, then such circuits are called dual circuits of each other.

Consider a series R-L-C circuit, the voltage equation can be written as:

$v(t) = R.i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$ ……………………………….. (1) 1 Mark

Consider a parallel R-L-C circuit, the current equation can be written as:

$i(t) = \frac{1}{R} v(t) + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt$ …………………………(2) 1 Mark



1 Mark

On comparing equations (1) & (2), it is seen that both the equations are integro-differential equations of similar kind. Therefore, the two circuits are dual circuits. The dual element pairs are:

Voltage source v(t) ⟵⟶ Current source i(t)
Resistance (R) ⟵⟶ Conductance (G = 1/R)
Inductance (L) ⟵⟶ Capacitance (C)
Series Circuit ⟵⟶ Parallel circuit

1 Mark

Examples of duality in electric circuit

- voltage – current
- parallel circuit – series circuit
- resistance – conductance
- voltage division – current division
- impedance – admittance
- capacitance – inductance
- reactance – susceptance
- short circuit – open circuit
- Kirchhoff's Voltage law – Kirchhoff's Current law
- Mesh – Node
- Thevenin's theorem – Norton's theorem

4 Attempt any THREE of the following. **12**

4 a) A series R-L-C circuit has R = 5Ω, L = 10mH and C = 15μF. Calculate:
(i) Resonant frequency
(ii) Q-factor of the circuit
(iii) Bandwidth
(iv) Voltage Magnification.

Ans:

Data Given:

R= 5 Ω, L=10mH = 10×10^{-3}H, C=15 μF = 15×10^{-6} F,

i) Resonant frequency:

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi}\sqrt{\frac{1}{10\times10^{-3}\times15\times10^{-6}}}$

= 410.94 Hz

1 Mark for each bit = 4 Marks

ii) Quality factor of circuit:

Q factor = $\frac{2\pi L f_r}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{2\pi \times 10 \times 10^{-3} \times 410.93}{5}$

= 5.16

iii) Bandwidth:

Bandwidth = $\frac{f_r}{Q\ factor}$

$= \frac{410.94}{5.16} =$ **79.64 Hz**

iv) Voltage Magnification:

Q factor = $\frac{1}{R}\sqrt{\frac{L}{C}}$

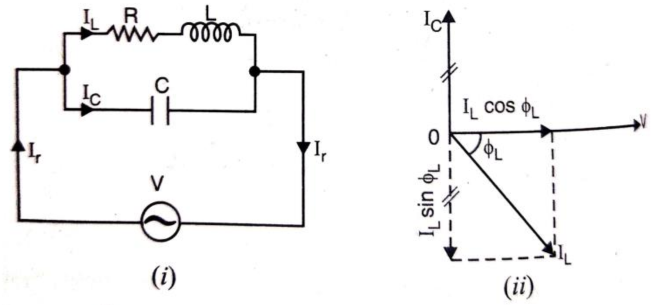
$= \frac{1}{5}\sqrt{\frac{10\times10^{-3}}{15\times10^{-6}}}$

= 5.16

4 b) Explain the "Current Magnification" in parallel resonant circuit consisting of inductive branch (RL) in parallel with a pure capacitor (C). Derive equation for it.

Ans:

Current Magnification in Parallel Resonant (RL||C) Circuit:



1 Mark for diagram

Current Magnification:

The Current Magnification or quality factor or Q-factor of parallel resonant circuit is defined as the ratio of the current circulating between two branches of the circuit to the current taken by the parallel circuit from the source. 1 Mark

Current Magnification = Q-factor = $\frac{Circulating\ current\ between\ L\ and\ C}{Input\ current\ from\ the\ source} = \frac{I_C}{I_r}$

At parallel resonance, the circulating current is I_C and circuit condition is,

$I_C - I_L \sin\emptyset_L = 0$

$I_C = I_L \sin\emptyset_L$

$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$

$Z_L{}^2 = X_L \times X_C = \omega_r L \frac{1}{\omega_r C} = \frac{L}{C}$ ………………….(1)

Total circuit input current, $I_r = I_L \cos\emptyset_L$

If the circuit impedance at resonance is Z_r, then

$I_r = \frac{V}{Z_r} = \frac{V}{Z_L} \times \frac{R}{Z_L}$

$\frac{1}{Z_r} = \frac{R}{Z_L{}^2}$

Substituting Z_L^2 from eq. (1),

2 Marks for stepwise derivation

$$\frac{1}{Z_r} = \frac{R}{\frac{L}{C}} = \frac{CR}{L}$$

$$Z_r = \frac{L}{CR} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Now, circulating current $I_C = V/X_C = \frac{V}{(\frac{1}{\omega_r C})} = \omega_r CV$

and input line current taken by circuit $I_r = \frac{V}{Z_r} = \frac{V}{\left[\frac{L}{CR}\right]} = \frac{VCR}{L}$

Current Magnification = Q-factor $= \frac{I_C}{I_r}$

Current Magnification = Q-factor $= \frac{\omega_r CV}{\frac{VCR}{L}} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$

The Q-factor of a parallel resonant circuit can also be expressed in term of L and C. Neglecting resistance R, the resonance frequency is given by;

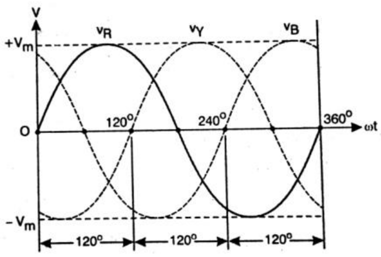
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Now,

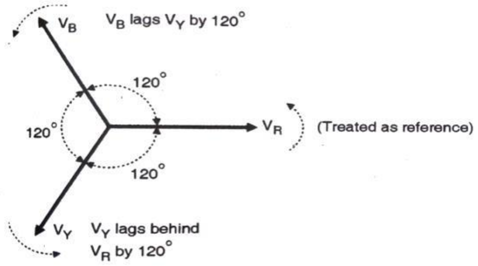
Current Magnification = Q- factor $= \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

4 c) Draw waveform of three-phase voltages. Draw phasor diagram for these voltages. Write equations for instantaneous values of these voltages. Express these voltages in polar form.

Ans:



1 Mark for waveform



1 Mark for phasor diagram

The equations of three-phase voltages can be represented by,

$v_R = V_m \sin\omega t$

$v_Y = V_m \sin(\omega t - 120^0)$

$v_B = V_m \sin(\omega t - 240^0) = V_m \sin(\omega t + 120^0)$

1 Mark for equations

Polar Form:

Let V be the RMS value of the phase voltage, $V = \frac{V_m}{\sqrt{2}}$

$V_R = V\angle 0°$

$V_Y = V\angle -120°$

$V_B = V\angle -240° = V\angle 120°$

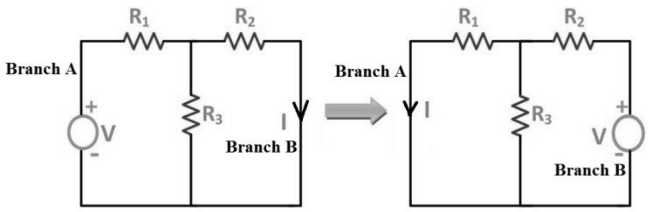
1 Mark for polar form

4 d) State and explain "Reciprocity theorem".

Ans:

Reciprocity theorem :

Reciprocity Theorem states that in any bilateral network if a voltage source V in one branch, say branch 'A', produces a current I in another branch, say branch 'B', then if the voltage source V is moved from the branch A to the branch B, it will cause the same current I in the first branch 'A', where the voltage source has been replaced by a short circuit. 2 Marks for statement



Steps for Solving a Network Utilizing Reciprocity Theorem:

Step 1: Firstly, select the branches, say A and B, between which reciprocity has to be established.

Step 2: The current I_1 in the branch B is obtained using any conventional network analysis method, when the source is in the branch A. 2 Marks for explanation

Step 3: The voltage source is moved to branch B.

Step 4: The current I_2 in the branch A, where the voltage source was existing earlier, is calculated.

Step 5: It is seen that the current I_1 obtained in the previous connection, i.e., in step 2 and the current I_2 which is calculated when the source is moved to branch B i.e. in step 4, are equal to each other.

The limitation of this theorem is that it is applicable only to single source networks and not in the multi-source network. The network where reciprocity theorem is applied should be linear and consist of resistors, inductors, capacitors and coupled circuits. The circuit should not have any time-varying elements.

5 Attempt any TWO of the following: 12

5 a) A coil having resistance of 5 Ω and an inductance of 0.2 H is connected in parallel with a series combination of 10 Ω resistor and 80 μF capacitor. If supply voltage is 230 V, 50 Hz, determine:

1) Total circuit impedance
2) Total current taken by the circuit
3) Power factor of the circuit
4) Branch currents
5) Power consumed by the circuit

Ans:

Data Given: Branch I: $R_1 = 5\ \Omega$ and $L = 0.2$ H

Branch II: $R_2 = 10\ \Omega$ and $C = 80\mu F = 80 \times 10^{-6}$ F

$V = 230V, \quad f = 50Hz$

(i) Total circuit impedance (Z):

Inductive reactance $X_L = 2\pi fL$

= 2 x π x 50 x 0.2 — ½ Mark for X_L

X_L = 62.83 Ω

Capacitive reactance $X_C = 1 / (2\pi fC)$

$X_C = 1 / (2\pi \times 50 \times 80 \times 10^{-6})$ — ½ Mark for X_C

X_C = 39.79 Ω

Branch 1 Impedance $Z_1 = (5 + j62.83)\ \Omega = \mathbf{63.03\ \angle 85.45°\Omega}$

Branch 2 Impedance $Z_2 = (10 - j39.79)\ \Omega = \mathbf{41.03\ \angle{-75.89}°\ \Omega}$

Since impedances are in parallel, total circuit impedance is given by,

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(\mathbf{63.03\ \angle 85.45°})(\mathbf{41.03\ \angle -75.89°})}{(5\ +\ j62.83) + (10 - j39.79)}$$

$$= \frac{\mathbf{2586.12\ \angle 9.56°}}{(15 + j23.04)} = \frac{\mathbf{2586.12\ \angle 9.56°}}{\mathbf{27.49\ \angle 56.93°}}$$ — 1 Mark for Z

∴**Total circuit impedance** $\mathbf{Z = 94.07\angle -47.37°}\Omega$

(ii) Total current (I) :

Total Current (I): $I = V / Z = \frac{230\angle 0^0}{94.07\angle -47.37°}$ — 1 Mark for I

$$= \mathbf{2.44\angle 47.37^0 A} = \ (\mathbf{1.65 + j\ 1.80})A$$

Angle between V and I is {0-47.37} = - 47.37° — 1 Mark for pf

(iii) Power factor of the circuit (cosϕ) :

cosϕ = cos(-47.37°) = **0.68 leading**

(iv) Branch Currents: — ½ Mark for each branch current = 1 Mark

Branch current $I_1 = V/Z_1 = \frac{230\angle 0^0}{63.03\ \angle 85.45°} = \mathbf{3.65\angle -85.45°\ A}$

Branch current $I_2 = V/Z_2 = \frac{230\angle 0^0}{41.03\ \angle -75.89°} = \mathbf{5.61\angle 75.89°\ A}$

(v) Power consumed by the circuit: — 1 Mark for P

P = V × I × cosϕ = 230 × 2.44 × 0.68

P = 381.62 watt

5 b) Using mesh analysis, find current in 5 Ω resistor in the network shown in Fig. 5(b).

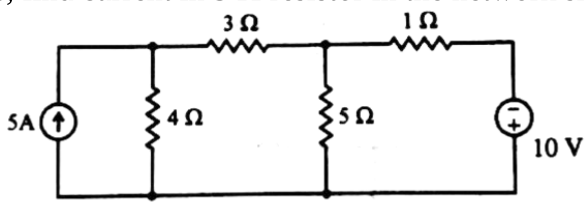


Fig. No. 5 (b)

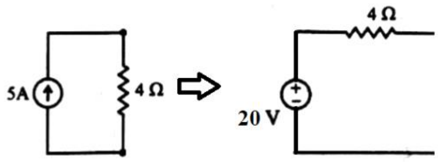
Ans:

A) Converting current source of 5A, 4Ω into equivalent voltage source:

Emf of voltage source V = I × R = 5×4= 20 volt

Internal resistance of voltage source = internal resistance of current source

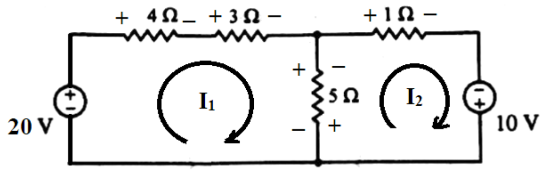
= 4 Ω



1 Mark

B) Modified circuit:

Replacing current source with equivalent voltage source, the modified circuit diagram is as shown below. The mesh currents can be marked as shown.



C) Mesh Analysis:

By applying KVL to Mesh 1:

$$20-(4+3)I_1-5(I_1-I_2)=0$$

$$20-12I_1+5I_2=0$$

$$12I_1-5I_2=20 \quad \ldots\ldots\ldots\ldots (1)$$

1 Mark for Eq. (1)

By applying KVL to Mesh 2:

$$10-5(I_2-I_1)-1I_2=0$$

$$10-6I_2+5I_1=0$$

$$-5I_1+6I_2=10 \ldots\ldots\ldots\ldots\ldots (2)$$

1 Mark for Eq. (2)

Expressing eq.(1) and (2) in matrix form,

$$\begin{bmatrix}12 & -5\\ -5 & 6\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}=\begin{bmatrix}20\\ 10\end{bmatrix}$$

1 Mark for Eq. in matrix form

$$\therefore \Delta=\begin{vmatrix}12 & -5\\ -5 & 6\end{vmatrix}=72-(25)=47$$

By Cramer's rule,

$$I_1=\frac{\begin{vmatrix}20 & -5\\ 10 & 6\end{vmatrix}}{\Delta}=\frac{(20\times 6)-10\times(-5)}{47}=\frac{120+50}{47}=\mathbf{3.62\ A}$$

½ Mark for I_1

$$I_2=\frac{\begin{vmatrix}12 & 20\\ -5 & 10\end{vmatrix}}{\Delta}=\frac{(12\times 10)-20\times(-5)}{47}=\frac{120+100}{47}=\mathbf{4.68\ A}$$

½ Mark for I_2

Current flowing through resistance of 5 Ω = $I_2 - I_1$ = 4.68 – 3.62
= 1.06 A in the direction of I_2

1 Mark

5 c) Find the current in 5 Ω resistor in the network shown in Fig. 5(c) by using Thevenin's theorem.

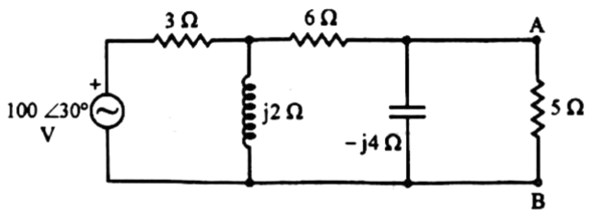
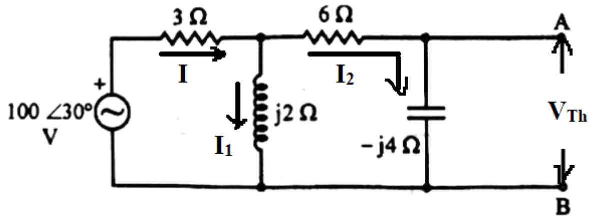


Fig. No. 5 (c)

Ans:

1) Determination of Thevenin's equivalent voltage V_{Th}:



2 Marks for stepwise calculation of V_{Th}

Thevenin's voltage V_{Th} is the open circuit voltage between load terminals A & B. It is seen that it is the voltage across capacitor.
The net impedance across 100V source is given by,

$Z = 3 + (j2)||(6\text{-}j4) = 3 + \frac{(j2)(6-j4)}{j2+6-j4} = 3 + \frac{8+j12}{6-j2} = 3 + \frac{14.42\angle 56.31°}{6.32\angle -18.43°}$

$= 3 + 2.28\angle 74.74° = 3 + 0.6 + j2.2 = \mathbf{(3.6 + j2.2)\Omega = 4.22\angle 31.43°\Omega}$

The total current $I = V/Z = \frac{100\angle 30°}{4.22\angle 31.43°} = 23.7\angle -1.43°$ A

The capacitor current $I_2 = I\frac{Z_1}{Z_1+Z_2} = (23.7\angle -1.43°)\frac{j2}{j2+6-j4}$

$= (23.7\angle -1.43°)\frac{2\angle 90°}{6.32\angle -18.43°}$

$= 7.5\angle 107° = (-2.2+j7.17)$ A

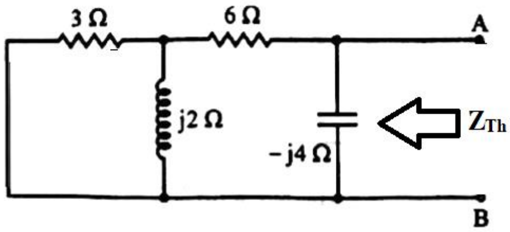
Thevenin's voltage $V_{Th} = (-j4)I_2 = (4\angle 90°)(7.5\angle 107°)$

$\mathbf{V_{Th} = 30\angle 197°}$ **volt = (-28.69 – j8.77) volt**

2) Determination of Thevenin's Equivalent Impedance Z_{Th}:

2 Marks for stepwise calculation of Z_{Th}

It is the impedance seen between the open circuited terminals A & B with all internal independent voltage sources replaced by short circuit and all internal independent current sources by open circuit.



$Z_{Th} = (-j4) \,||\, \{6 + (3||j2)\}$

$= (-j4) \,||\, \{6 + (\frac{3\times 2\angle 90°}{3+j2})\} = (-j4) \,||\, \{6 + (\frac{6\angle 90°}{3.606\angle 33.7°})\}$

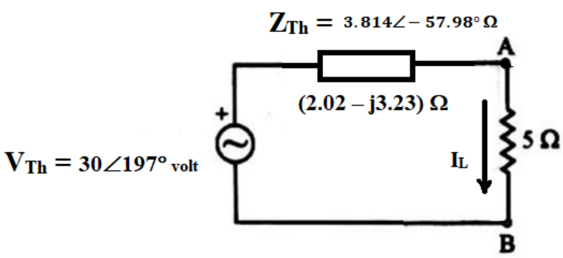
$= (-j4) \,||\, \{6 + (1.66\angle 56.3°)\} = (-j4) \,||\, \{6 + 0.92 + j1.38\}$

$= (-j4) \,||\, \{6.92 + j1.38\} = (-j4) \,||\, \{7.056\angle 11.28°\}$

$= \frac{(4\angle -90°)(7.056\angle 11.28°)}{(-j4+6.92+j1.38)} = \frac{28.224\angle -78.72°}{7.4\angle -20.74°}$

$\mathbf{Z_{Th} = 3.814\angle -57.98°\,\Omega = (2.02 - j3.23)\,\Omega}$

3) Thevenin's Equivalent Circuit:



1 Mark for Thevenin's Eq. circuit

1 Mark for I_L

The current in 5Ω resistor is given by,

$I_L = \frac{V_{Th}}{(Z_{Th}+R_L)} = \frac{30\angle 197°}{(\mathbf{2.02 - j3.23+5})} = \frac{30\angle 197°}{(\mathbf{7.02 - j3.23})} = \frac{30\angle 197°}{7.73\angle -24.71°}$

$\mathbf{I_L = 3.88\angle 221.71°\ A}$

6 Attempt any TWO of the following: 12

6 a) For a series R-L-C circuit consisting of R = 5Ω, L = 0.01 H and C = 10 μF supplied with 230 V, 50 Hz supply, determine:

i) Circuit impedance
ii) Circuit current
iii) Circuit power factor
iv) Active power
v) Reactive power
vi) Apparent power

Ans:

Data Given: R = 5 Ω, L = 0.01 H, C = 10μF = 10×10^{-6}F
V = 230V, f = 50 Hz

1 Mark for each bit = 6 Marks

(i) Circuit Impedance:

$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.01 = 3.142\ \Omega$

$X_c = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times50\times10\times10^{-6}} = 318.31\ \Omega$

$Z = R + j(X_L - X_C) = 5 + j(3.142 - 318.31)$

Z = (5 – j 315.168) Ω = 315.21∠-89.1° Ω

(ii) Circuit current:

Circuit current $I = \frac{V}{Z} = \frac{230\angle 0°}{315.21\angle -89.1°}$ = **0.73∠89.1° A**

(iii) Circuit power factor:

Circuit Power factor angle ϕ = 89.1° leading

Circuit power factor cosϕ = cos(89.1°) = **0.016 (leading)**

(iv) Active Power (P):

P = VI cos ϕ = 230 × 0.73 × 0.016
= **2.6864 W**

(v) Reactive Power (Q):

Q = VI sin ϕ = 230 × 0.73 × sin(89.1°)
= **167.88 VAR**

(vi) Apparent Power (S):

Apparent Power = S = VI = 230 × 0.73 = 167.9 VA

6 b) A star connected capacitive load is supplied from 3ϕ, 415V, 50Hz supply. If the line current is 15 A and total 3ϕ power taken from supply is 30 kW, find:

(i) Power factor
(ii) Resistance in each phase.
(iii) Capacitance in each phase

Ans:

Data Given: V_L = 415V, f = 50Hz, I_L = 15A, P = 30 kW = 30000 W

In Star connection,

$V_L = \sqrt{3} \times V_{Ph}$ and $I_L = I_{Ph}$

Therefore, $V_{Ph} = V_L / \sqrt{3} = 415 / \sqrt{3}$ = **239.6 Volt.**

And $I_L = I_{Ph}$ = **15Amp.**

∴ Impedance per phase, $Z_{Ph} = V_{Ph} / I_{Ph} = 239.6 / 15$

Z_{Ph}= 15.97 Ω — 1 Mark for Z_{Ph}

i) Power factor:

Total three-phase power is given by,

$P = 3V_{Ph} I_{Ph} \cos\phi$ **Or** $P = \sqrt{3} V_L I_L \cos\phi$

$30 \times 10^3 = 3 \times 239.6 \times 15 \times \cos\phi$

Therefore,

$\cos\phi = 30 \times 10^3 / (3 \times 239.6 \times 15)$

∴ cosϕ = **2.78 !!!!!!!!!!**

Since maximum value of cosϕ = 1, here is data mismatch !!!!!!!!!!!

Assuming total 3ϕ power as 3 kW instead of 30 kW,

$\cos\phi = 3 \times 10^3 / (3 \times 239.6 \times 15)$

cosϕ = 0.278 leading — 1 Mark for cosϕ

$\phi = \cos^{-1}(0.278)$ = **73.84°** — 1 Mark for ϕ

(NOTE: Examiner is requested to award appropriate marks to the student for any other suitable assumption of data and if attempted to solve)

ii) Resistance in each phase:

Resistance per phase (R_{ph}) = $Z_{ph} \times \cos\phi$ = 15.97 x 0.278

R_{ph} = 4.44 Ω — 1 Mark for R_{Ph}

iii) Reactance in each phase:

Reactance per phase (X_{ph}) = $Z_{ph} \times \sin\phi$ = 15.97 x sin(73.84°)

X_{ph} = 15.34 Ω — 1 Mark for X_{Ph}

Since capacitive reactance $X_C = X_{ph} = \frac{1}{2\pi f C}$

Capacitance in each phase $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(15.34)} = \mathbf{207.5 \times 10^{-6}}$ **F** — 1 Mark for C

6 c) Determine the voltage 'V' across 5 Ω resistor in the network shown in Fig. 6(c) using superposition theorem.

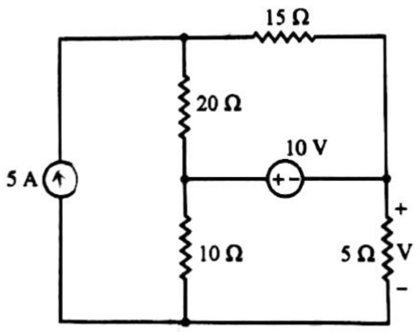
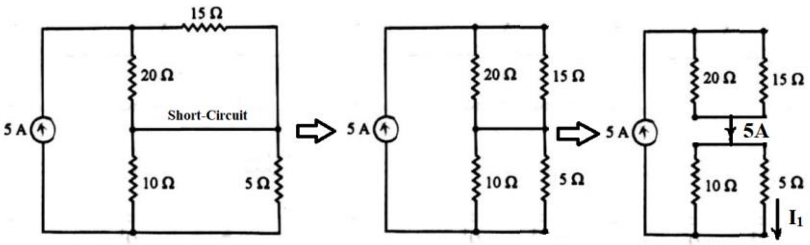


Fig. No. 6 (c)

Ans:

(A) Consider current source of 5A acting alone:

The 10V source is replaced by short-circuit (S.C.)



1 Mark for diagram

1 Mark for I_1

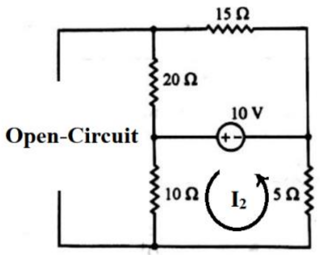
The total source current of 5A is divided and then flows through 5Ω & 10Ω.

The current flowing through 5Ω is given by current division formula as,

$I_1 = 5\times\{10/(10+5)\}$ = **3.33A (Downward)**

(B) Consider voltage source of 10V acting alone:

The 5A source is replaced by open-circuit (O.C.)



1 Mark for diagram

1 Mark for I_2

The current in lower mesh and flowing through 5Ω is given by,

$I_2 = 10/(10+5)$ = **0.67 A (Upward)**

(C) Total current in 5Ω resistor:

By Superposition theorem, the current through 5Ω due to both sources, assuming downward current positive and upward current negative, is given by,

I = I_1-I_2= (3.33- 0.67) = **2.66 A (Downward)** — 1 Mark for I

Voltage across 5Ω resistor is given by,

V = 5(I) = 5(2.66) = 13.3 volt — 1 Mark for V

V = 13.3 volt