



SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code:

17301

**Important Instructions to examiners:**

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		<b>Attempt any <u>TEN</u> of the following:</b>	<b>20</b>
	a)	Find the gradient of the curve $y = \sqrt{x^3}$ at $x = 4$	<b>02</b>
	Ans	$y = \sqrt{x^3}$ $\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x^3}} \cdot 3x^2$ at $x = 4$ $\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{(4)^3}} \cdot 3(4)^2 = 3$ OR $y = \sqrt{x^3}$ $\therefore y = x^{3/2}$ $\therefore \frac{dy}{dx} = \frac{3}{2} x^{1/2}$ at $x = 4$ $\therefore \frac{dy}{dx} = \frac{3}{2} (4)^{1/2} = 3$	1  1
	b)	Divide 100 into two parts such that their product is maximum.	<b>02</b>
	Ans	Let $x$ and $y$ be two parts of 100 $\therefore x + y = 100$ $\therefore y = 100 - x$ Product $P = xy$	



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1.	b)	$\therefore P = x(100 - x)$ $\therefore P = 100x - x^2$ $\frac{dP}{dx} = 100 - 2x$ $\therefore \frac{d^2P}{dx^2} = -2 \quad \therefore \text{Product is maximum.}$ Let $\frac{dP}{dx} = 0$ $\therefore 100 - 2x = 0$ $\therefore x = 50 \text{ and } y = 50$	$\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$
	c)	Evaluate: $\int \frac{x^2}{4+x^2} dx$	<b>02</b>
	Ans	$\int \frac{x^2}{4+x^2} dx$ $= \int \frac{4+x^2-4}{4+x^2} dx$ $= \int \left(1 - \frac{4}{4+x^2}\right) dx$ $= \int \left(1 - \frac{4}{2^2+x^2}\right) dx$ $= x - \frac{4}{2} \tan^{-1} \frac{x}{2} + c$ $= x - 2 \tan^{-1} \frac{x}{2} + c$	1     $\frac{1}{2} + \frac{1}{2}$
d)	Evaluate: $\int \log x dx$	<b>02</b>	
Ans	$\int \log x dx$ $= \int \log x \cdot 1 dx$ $= \log x \int 1 dx - \int \left( \int 1 dx \cdot \frac{d(\log x)}{dx} \right) dx$ $= x \cdot \log x - \int x \cdot \frac{1}{x} dx$ $= x \cdot \log x - \int 1 dx$ $= x \cdot \log x - x + c$	$\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$	



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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	Evaluate: $\int \tan^3 x dx$	<b>02</b>
	Ans	$\begin{aligned} \text{Let } \int \tan^3 x dx \\ &= \int \tan^2 x \cdot \tan x dx \\ &= \int (\sec^2 x - 1) \cdot \tan x dx \\ &= \int (\sec^2 x \cdot \tan x - \tan x) dx \\ &= \int \sec^2 x \cdot \tan x dx - \int \tan x dx \\ &= \int \sec^2 x \cdot \tan x dx - \log(\sec x) + c \\ \text{Put } \tan x &= t \\ \therefore \sec^2 x dx &= dt \\ &= \int t dt - \log(\sec x) + c \\ &= \frac{t^2}{2} - \log(\sec x) + c \\ &= \frac{\tan^2 x}{2} - \log(\sec x) + c \end{aligned}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
	f)	Evaluate: $\int \frac{dx}{(x+1)(x+2)}$	<b>02</b>
	Ans	$\begin{aligned} \frac{1}{(x+1)(x+2)} &= \frac{A}{x+1} + \frac{B}{x+2} \\ \therefore 1 &= A(x+2) + B(x+1) \\ \text{Put } x &= -1 \\ \therefore A &= 1 \\ \text{Put } x &= -2 \\ \therefore B &= -1 \\ \therefore \frac{1}{(x+1)(x+2)} &= \frac{1}{x+1} + \frac{-1}{x+2} \\ \therefore \int \frac{1}{(x+1)(x+2)} dx &= \int \frac{1}{x+1} + \frac{-1}{x+2} dx \\ &= \log(x+1) - \log(x+2) + c \\ &= \frac{\log(x+1)}{\log(x+2)} + c \end{aligned}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p>



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1.	f)	<p>OR</p> $\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x^2 + 3x + 2}$ $T.T. = \frac{9}{4}$ $\therefore \int \frac{dx}{x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2}$ $= \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \frac{1}{4}}$ $= \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \frac{1}{2^2}}$ $= \frac{1}{2 \times \frac{1}{2}} \log \left  \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right  + c$ $= \log \left  \frac{x+1}{x+2} \right  + c$	<p>½</p> <p>½</p> <p>1</p>
	g)	<p>Evaluate: <math>\int \frac{\sin x}{\sin 2x} dx</math></p>	02
	Ans	$\int \frac{\sin x}{\sin 2x} dx$ $= \int \frac{\sin x}{2 \sin x \cos x} dx$ $= \frac{1}{2} \int \frac{1}{\cos x} dx$ $= \frac{1}{2} \int \sec x dx$ $= \frac{1}{2} \log (\sec x + \tan x) + c$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
h)	<p>If <math>\int_0^a 3x^2 dx = 8</math> find the value of 'a'.</p>	02	
Ans	$\int_0^a 3x^2 dx = 8$		



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1.	h)	$\left[ 3 \frac{x^3}{3} \right]_0^a = 8$ $\left[ x^3 \right]_0^a = 8$ $\therefore a^3 - 0 = 8$ $\therefore a^3 = 8$ $\therefore a = 2$	1   ½  ½
	i)	Find the area bounded $y = x^2 - 9$ , $x = 0$ to $x = 3$ and the X-axis	02
	Ans	$\text{Area } A = \int_a^b y dx$ $= \int_0^3 (x^2 - 9) dx$ $= \left[ \frac{x^3}{3} - 9x \right]_0^3$ $= \left[ \frac{3^3}{3} - 9(3) \right] - \left[ \frac{0^3}{3} - 9(0) \right]$ $= -18 \text{ i.e. } 18$	½  ½  ½  ½
j)	Find order and degree of the differential equation $\left( \frac{d^2 y}{dx^2} \right)^{\frac{2}{3}} = \sqrt{\left( y + \frac{dy}{dx} \right)}$		02
Ans	$\left( \frac{d^2 y}{dx^2} \right)^{\frac{2}{3}} = \sqrt{\left( y + \frac{dy}{dx} \right)}$ $\therefore \left( \frac{d^2 y}{dx^2} \right)^{\frac{2}{3}} = \left( y + \frac{dy}{dx} \right)^{\frac{1}{2}}$ <p>Taking 6<sup>th</sup> power on both sides,</p> $\left( \frac{d^2 y}{dx^2} \right)^4 = \left( y + \frac{dy}{dx} \right)^3$ $\therefore \text{Order} = 2$ $\text{Degree} = 4$	1  1	



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1.	(k)	Form a D.E. if $y = a \cos(x + b)$	<b>02</b>
	Ans	$y = a \cos(x + b)$ $\frac{dy}{dx} = -a \sin(x + b)$ $\frac{d^2y}{dx^2} = -a \cos(x + b)$ $\frac{d^2y}{dx^2} = -y$ $\therefore \frac{d^2y}{dx^2} + y = 0$	$\frac{1}{2}$ $\frac{1}{2}$ <b>1</b>
	l)	Verify that $y = 4 \sin 3x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y = 0$	<b>02</b>
	Ans	$y = 4 \sin 3x$ $\therefore \frac{dy}{dx} = 12 \cos 3x$ $\therefore \frac{d^2y}{dx^2} = -36 \sin 3x$ $\frac{d^2y}{dx^2} = -9(4 \sin 3x)$ $\frac{d^2y}{dx^2} = -9y$ $\therefore \frac{d^2y}{dx^2} + 9y = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
m)	Find the probability of occurrence of the digit 3 when an unbiased dice is thrown.		<b>02</b>
	Ans	$S = \{1, 2, 3, 4, 5, 6\}$ $\therefore n(S) = 6$ $A = \{3\}$ $\therefore n(A) = 1$ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{6} = 0.1667$	$\frac{1}{2}$ $\frac{1}{2}$ <b>1</b>
n)	A coin is tossed 3 times. What is the probability that appears an odd number of times?		<b>02</b>
		$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$	



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<b>1.</b>	n)	$\therefore n(S) = 8$	$\frac{1}{2}$
	Ans	<p><math>A</math> : head appears odd number of times</p> <p><math>A = \{HHH, THH, THT, HTT\} \quad \therefore n(A) = 4</math></p> <p><math>\therefore p(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = 0.5</math></p> <p>OR</p> <p><math>\therefore n(S) = 8</math></p> <p><math>A</math> : tail appears odd number of times</p> <p><math>A = \{HHT, HTH, THH, TTT\} \quad \therefore n(A) = 4</math></p> <p><math>\therefore p(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = 0.5</math></p> <p>(Note: If student has considered either head or tail (and attempted to solve give appropriate marks )</p>	<p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>
<b>2.</b>		<p><b>Attempt any <u>FOUR</u> of the following:</b></p>	<b>16</b>
	a)	Determine a and b such that slope of curve $2y^3 = ax^2 + b$ at $(1, -1)$ is same as the slope of $x + y = 0$	<b>04</b>
	Ans	<p><math>2y^3 = ax^2 + b</math></p> <p><math>\therefore 6y^2 \frac{dy}{dx} = 2ax</math></p> <p><math>\therefore \frac{dy}{dx} = \frac{2ax}{6y^2} = \frac{ax}{3y^2}</math></p> <p>at <math>(1, -1)</math></p> <p><math>\therefore \frac{dy}{dx} = \frac{a(1)}{3(-1)^2} = \frac{a}{3}</math></p> <p><math>\therefore x + y = 0</math></p> <p><math>\therefore 1 + \frac{dy}{dx} = 0</math></p> <p><math>\therefore</math> Slope is <math>\frac{dy}{dx} = -1</math></p> <p><math>\therefore</math> slopes are equal.</p> <p><math>\therefore -1 = \frac{a}{3}</math></p> <p><math>\therefore a = -3</math> and <math>b = 1</math></p>	<p>1</p> <p>1</p> <p>1</p>
			$\frac{1}{2} + \frac{1}{2}$



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2.	b)	Find the maximum and minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$	<b>04</b>
	Ans	Let $y = 2x^3 - 21x^2 + 36x - 20$ $\therefore \frac{dy}{dx} = 6x^2 - 42x + 36$ $\therefore \frac{d^2y}{dx^2} = 12x - 42$ Consider $\frac{dy}{dx} = 0$ $6x^2 - 42x + 36 = 0$ $\therefore x = 6$ or $x = 1$ at $x = 6$ $\frac{d^2y}{dx^2} = 12(6) - 42 = 30 > 0$ $\therefore y$ is minimum at $x = 6$ $y_{\min} = 2(6)^3 - 21(6)^2 + 36(6) - 20$ $= -128$ at $x = 1$ $\frac{d^2y}{dx^2} = 12(1) - 42 = -30 < 0$ $\therefore y$ is maximum at $x = 1$ $y_{\max} = 2(1)^3 - 21(1)^2 + 36(1) - 20$ $= -3$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	c)	Find radius of curvature of $y = \log(\sin x)$ at $x = \frac{\pi}{2}$	<b>04</b>
	Ans	$y = \log(\sin x)$ $\therefore \frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$ $\therefore \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$ at $x = \frac{\pi}{2}$ $\frac{dy}{dx} = \cot \frac{\pi}{2} = 0$ $\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 \frac{\pi}{2} = -1$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$





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2	c)	$\therefore \text{Radius of curvature is, } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $\therefore \rho = \frac{\left[1 + (0)^2\right]^{\frac{3}{2}}}{-1}$ $\therefore \rho = -1 \text{ i.e. } 1$	1 1
	d)	Evaluate $\int \sin^{-1} x dx$	<b>04</b>
	Ans	$\int \sin^{-1} x \cdot 1 dx$ $= \sin^{-1} x \int 1 dx - \int \left( \int 1 dx \frac{d}{dx} \sin^{-1} x \right) dx$ $= \sin^{-1} x \cdot x - \int \frac{x}{\sqrt{1-x^2}} dx$ $= x \sin^{-1} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx$ $= x \sin^{-1} x + \frac{1}{2} 2\sqrt{1-x^2} + c$ $= x \sin^{-1} x + \sqrt{1-x^2} + c$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1
e)	Evaluate: $\int \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)(3 + \sin x)}$	<b>04</b>	
Ans	Put $\sin x = t$ $\therefore \cos x dx = dt$ $= \int \frac{1}{(1+t)(2+t)(3+t)} dt$ consider $\frac{1}{(1+t)(2+t)(3+t)} = \frac{A}{1+t} + \frac{B}{2+t} + \frac{C}{3+t}$ $1 = (2+t)(3+t)A + (1+t)(3+t)B + (1+t)(2+t)C$ Put $t = -1$ $\therefore A = \frac{1}{2}$ Put $t = -2$ $\therefore B = -1$ Put $t = -3$	$\frac{1}{2}$          $\frac{1}{2}$	



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2.	e)	$\therefore C = \frac{1}{2}$ $\frac{1}{(1+t)(2+t)(3+t)} = \frac{\frac{1}{2}}{1+t} + \frac{-1}{2+t} + \frac{\frac{1}{2}}{3+t}$ $\int \frac{1}{(1+t)(2+t)(3+t)} dt = \int \left( \frac{\frac{1}{2}}{1+t} + \frac{-1}{2+t} + \frac{\frac{1}{2}}{3+t} \right) dt$ $= \frac{1}{2} \log(1+t) - \log(2+t) + \frac{1}{2} \log(3+t) + c$ $= \frac{1}{2} \log(1 + \sin x) - \log(2 + \sin x) + \frac{1}{2} \log(3 + \sin x) + c$	<p>½</p> <p>½+½+½</p> <p>½</p>
	f)	Evaluate: $\int \frac{dx}{1+2(x+2)^2}$	04
	Ans	$\int \frac{dx}{1+2(x+2)^2}$ $= \int \frac{dx}{1+2(x^2+4x+4)}$ $= \int \frac{dx}{1+2x^2+8x+8}$ $= \int \frac{dx}{2x^2+8x+9}$ $= \frac{1}{2} \int \frac{dx}{x^2+4x+\frac{9}{2}}$ $T.T = \left( \frac{1}{2} \times 4 \right)^2 = 4$ $= \frac{1}{2} \int \frac{dx}{x^2+4x+4+\frac{9}{2}-4}$ $= \frac{1}{2} \int \frac{dx}{(x+2)^2 + \frac{1}{2}}$ $= \frac{1}{2} \int \frac{dx}{(x+2)^2 + \left( \frac{1}{\sqrt{2}} \right)^2}$	<p>½</p> <p>½</p> <p>½</p> <p>1</p> <p>½</p>



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2.	f)	$= \frac{1}{2} \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1} \left( \frac{x+2}{\frac{1}{\sqrt{2}}} \right) + c$ $= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2}(x+2)) + c$ <p>OR</p> $\int \frac{dx}{1+2(x+2)^2}$ <p>Put <math>x+2 = t</math>  <math>\therefore dx = dt</math></p> $= \int \frac{dt}{1+2t^2}$ $= \frac{1}{2} \int \frac{dt}{\frac{1}{2}+t^2}$ $= \frac{1}{2} \int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2 + t^2}$ $= \frac{1}{2} \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1} \left( \frac{t}{\frac{1}{\sqrt{2}}} \right) + c$ $= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2}t) + c$ $= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2}(x+2)) + c$ <p>OR</p> $\int \frac{dx}{1+2(x+2)^2}$ $= \frac{1}{2} \int \frac{dx}{\frac{1}{2}+(x+2)^2}$ $= \frac{1}{2} \int \frac{dx}{\left(\frac{1}{\sqrt{2}}\right)^2+(x+2)^2}$	<p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p>



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2.		$= \frac{1}{2} \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1} \left( \frac{x+2}{\frac{1}{\sqrt{2}}} \right) + c$ $= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2}(x+2)) + c$ <p>OR</p> $\int \frac{dx}{1+2(x+2)^2} = \int \frac{dx}{1^2 + (\sqrt{2}(x+2))^2}$ $= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2}(x+2)) + c$	2
3.		<p>Solve any <b>FOUR</b> of the following:</p> <p>a) Evaluate: <math>\int_0^1 \frac{dx}{1-x+x^2}</math></p> <p>Ans <math>\int_0^1 \frac{dx}{1-x+x^2} = \int_0^1 \frac{dx}{x^2-x+1}</math></p> $T.T. = \left( \frac{1}{2} \times (-1) \right)^2 = \frac{1}{4}$ $\int_0^1 \frac{dx}{x^2 - x + \frac{1}{4} + 1 - \frac{1}{4}}$ $= \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$ $= \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= \left[ \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left( \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right]_0^1 = \left[ \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x-1}{\sqrt{3}} \right) \right]_0^1$ $= \left[ \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2(1)-1}{\sqrt{3}} \right) \right] - \left[ \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2(0)-1}{\sqrt{3}} \right) \right]$	16
			04
			1
			½
			1
			½



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3.	a)	$= \frac{2\pi}{3\sqrt{3}}$	1
	b)	<p>-----</p> <p>Evaluate: <math>\int_0^{\frac{\pi}{2}} \frac{dx}{9+16\cos^2 x}</math></p>	04
	Ans	$\int_0^{\frac{\pi}{2}} \frac{dx}{9+16\cos^2 x}$ $= \int_0^{\frac{\pi}{2}} \frac{dx / \cos^2 x}{9+16\cos^2 x \cos^2 x}$ $= \int_0^{\frac{\pi}{2}} \frac{dx / \cos^2 x}{\frac{9}{\cos^2 x} + \frac{16\cos^2 x}{\cos^2 x}}$ $= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9\sec^2 x + 16}$ $= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9(1 + \tan^2 x) + 16}$ $= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9 + 9\tan^2 x + 16}$ $= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9\tan^2 x + 25}$ <p>Put <math>\tan x = t</math>  <math>\sec^2 x dx = dt</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>when <math>x \rightarrow 0</math> to <math>\frac{\pi}{2}</math>  <math>t \rightarrow 0</math> to <math>\infty</math></p> </div> $= \int_0^{\infty} \frac{dt}{9t^2 + 25}$ $= \frac{1}{9} \int_0^{\infty} \frac{dt}{t^2 + \frac{25}{9}}$	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
		<p>OR</p> $\int_0^{\infty} \frac{dt}{9t^2 + 25} = \int_0^{\infty} \frac{dt}{(3t)^2 + 5^2}$	½



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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$= \frac{1}{9} \int_0^{\infty} \frac{dt}{t^2 + \left(\frac{5}{3}\right)^2}$ $= \left[ \frac{1}{9} \times \frac{1}{\frac{5}{3}} \tan^{-1} \left( \frac{t}{\frac{5}{3}} \right) \right]_0^{\infty} \quad \text{OR} \quad \left[ \frac{1}{5 \times 3} \tan^{-1} \left( \frac{3t}{5} \right) \right]_0^{\infty}$ $= \left[ \frac{1}{15} \tan^{-1} \left( \frac{3t}{5} \right) \right]_0^{\infty}$ $= \left[ \frac{1}{15} \tan^{-1} \left( \frac{3(\infty)}{5} \right) \right] - \left[ \frac{1}{15} \tan^{-1} \left( \frac{3(0)}{5} \right) \right]$ $= \left[ \frac{1}{15} \tan^{-1}(\infty) \right] - \left[ \frac{1}{15} \tan^{-1}(0) \right]$ $= \frac{1}{15} \times \frac{\pi}{2}$ $= \frac{\pi}{30}$	<p><math>\frac{1}{2}</math></p> <p>1</p>
	c) Ans	<p>Find the area included between the curves <math>y^2 = 4ax</math> and <math>x^2 = 4ay</math></p> <p><math>y^2 = 4ax</math> ----- (1)</p> <p><math>x^2 = 4ay</math></p> <p><math>\therefore y = \frac{x^2}{4a}</math></p> <p><math>\therefore \text{eq}^n \cdot (1) \Rightarrow \left( \frac{x^2}{4a} \right)^2 = 4ax</math></p> <p><math>\frac{x^4}{16a^2} = 4ax</math></p> <p><math>\therefore x^4 = 64a^3x</math></p> <p><math>\therefore x^4 - 64a^3x = 0</math></p> <p><math>\therefore x(x^3 - 64a^3) = 0</math></p> <p><math>\therefore x = 0, 4a</math></p> <p>Area <math>A = \int_a^b (y_1 - y_2) dx</math></p> <p><math>\therefore A = \int_0^{4a} \left( 2\sqrt{a}\sqrt{x} - \frac{x^2}{4a} \right) dx</math></p>	<p><b>04</b></p> <p>1</p> <p>1</p>



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3.	c)	$\therefore A = \int_0^{4a} \left( 2\sqrt{ax^2} - \frac{x^2}{4a} \right) dx$ $\therefore A = \left( \frac{2\sqrt{ax^2}^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12a} \right)_0^{4a}$ $\therefore A = \left( \frac{2\sqrt{a}(4a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(4a)^3}{12a} \right) - 0$ $\therefore A = \frac{16a^2}{3} \text{ or } 5.333a^2$	<p>1</p> <p>½</p> <p>½</p>
	d)	<p>Solve <math>\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0</math></p> <p>Ans <math>\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0</math></p> <p><math>\sec^2 x \tan y dx = -\sec^2 y \tan x dy</math></p> <p><math>\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy</math></p> <p><math>\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy</math></p> <p><math>\log(\tan x) = -\log(\tan y) + c</math></p>	<p>04</p> <p>1</p> <p>1</p> <p>1+1</p>
	e)	<p>Solve: <math>\sin^{-1}\left(\frac{dy}{dx}\right) = x + y</math></p> <p>Ans <math>\sin^{-1}\left(\frac{dy}{dx}\right) = x + y</math></p> <p><math>\frac{dy}{dx} = \sin(x + y) \text{ -----(1)}</math></p> <p>Put <math>x + y = v</math></p> <p><math>\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}</math></p> <p><math>\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1</math></p> <p>From (1)</p> <p><math>\frac{dv}{dx} - 1 = \sin v</math></p>	<p>04</p> <p>1</p> <p>½</p>



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3.	e)	$\therefore \frac{dv}{dx} = \sin v + 1$ $\therefore \frac{dv}{\sin v + 1} = dx$ $\therefore \int \frac{dv}{1 + \sin v} = \int dx$ $\int \frac{1}{1 + \sin v} \times \frac{1 - \sin v}{1 - \sin v} dv = \int dx$ $\int \frac{1 - \sin v}{1 - \sin^2 v} dv = x + c$ $\int \frac{1 - \sin v}{\cos^2 v} dv = x + c$ $\int \sec^2 v - \tan v \sec v dv = x + c$ $\tan v - \sec v = x + c$ $\tan(x + y) - \sec(x + y) = x + c$ <p>OR</p> $\therefore \int \frac{dv}{1 + \sin v} = \int dx$ <p>Put <math>\tan \frac{v}{2} = t</math>, <math>dv = \frac{2dt}{1+t^2}</math>, <math>\sin v = \frac{2t}{1+t^2}</math></p> $\therefore \int \frac{\frac{2dt}{1+t^2}}{1 + \left(\frac{2t}{1+t^2}\right)} = x + c$ $\therefore \int \frac{2dt}{1+t^2+2t} = x + c$ $\therefore \int \frac{2dt}{(t+1)^2} = x + c$ $\therefore \int 2(t+1)^{-2} dt = x + c$ $\therefore 2 \frac{(t+1)^{-1}}{-1} = x + c \quad \therefore \frac{-2}{(t+1)} = x + c$ $\frac{-2}{\left(\tan \frac{(x+y)}{2} + 1\right)} = x + c$ <p>OR</p> $\therefore \int \frac{dv}{1 + \sin v} = \int dx$ $\therefore \int \frac{dv}{1 + \cos\left(\frac{\pi}{2} - v\right)} = x + c$	<p>1</p> <p>1</p> <p>½</p> <p>1</p> <p>1</p> <p>1</p> <p>½</p> <p>1</p>







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3.	f)	$\therefore -\log\left(1 + \frac{y^2}{x^2}\right) = \log x + c$	$\frac{1}{2}$
-----			
4.		<b>Attempt any <u>FOUR</u> of the following:</b>	<b>16</b>
	a)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$	<b>04</b>
	Ans	Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ -----(1)	
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$	
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ -----(2)	1
		add (1) and (2)	
		$\therefore I + I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$	1
		$\therefore 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$	
		$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 dx$	
		$\therefore 2I = [x]_0^{\frac{\pi}{2}}$	$\frac{1}{2}$
		$\therefore 2I = \frac{\pi}{2} - 0$	1
		$\therefore I = \frac{\pi}{4}$	$\frac{1}{2}$
-----			
	b)	Evaluate: $\int_0^{\pi} \frac{dx}{5 + 4 \cos x}$	<b>04</b>
	Ans	Put $\tan \frac{x}{2} = t$	



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4.	b)	$\cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px 0;">                     when <math>x \rightarrow 0</math> to <math>\pi</math>  <math>t \rightarrow 0</math> to <math>\infty</math> </div> $\therefore I = \int_0^{\infty} \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \frac{2dt}{1+t^2}$ $\therefore I = 2 \int_0^{\infty} \frac{1}{5(1+t^2)+4(1-t^2)} dt$ $\therefore I = 2 \int_0^{\infty} \frac{1}{5+5t^2+4-4t^2} dt$ $\therefore I = 2 \int_0^{\infty} \frac{1}{t^2+9} dt$ $\therefore I = 2 \int_0^{\infty} \frac{1}{t^2+(3^2)} dt$ $\therefore I = \frac{2}{3} \left[ \tan^{-1} \left( \frac{t}{3} \right) \right]_0^{\infty}$ $\therefore I = \frac{2}{3} \left[ \tan^{-1} \left( \frac{\infty}{3} \right) - \tan^{-1} \left( \frac{0}{3} \right) \right]$ $\therefore I = \frac{2}{3} \left[ \tan^{-1}(\infty) - \tan^{-1}(0) \right]$ $\therefore I = \frac{2}{3} \frac{\pi}{2} = \frac{\pi}{3} \text{ or } 60$	<p>1</p> <p>½</p> <p>½</p> <p>1</p> <p>½</p> <p>½</p>
	c)	Find the area of the circle $x^2 + y^2 = 9$ using integration.	<b>04</b>
	Ans	$x^2 + y^2 = 9$ $\therefore y^2 = 9 - x^2$ $\therefore y = \sqrt{9 - x^2} = \sqrt{3^2 - x^2}$ $\therefore A = 4 \int_a^b y dx$ $= 4 \int_0^3 \sqrt{3^2 - x^2} dx$ $= 4 \left[ \frac{x}{2} \sqrt{3^2 - x^2} + \frac{3^2}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^3$	<p>1</p> <p>1</p>







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4.	f)	$\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \frac{1}{2\sqrt{1-x^2}} (-2x) = m \frac{dy}{dx}$ <p>Multiply by <math>\sqrt{1-x^2}</math></p> $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = m \frac{dy}{dx} \sqrt{1-x^2}$ $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = m^2 y \quad \text{from (1)}$ $\therefore (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$ <p>(Note: If student has considered -1 at index place ) (and attempted to solve give appropriate marks )</p>	<p>1</p> <p>1</p> <p>1</p> <p>½</p>
5.	a)	<p><b>Attempt any <u>FOUR</u> of the following:</b></p> <p>Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on two dice is neither a multiple of 2 nor a multiple of 3.</p> <p>Ans</p> $S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$ $(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$ $(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$ $(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$ $(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$ $(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$ $\therefore n(S) = 36$ <p>A = multiple of 2 or multiple of 3</p> $A = \{(1,1)(1,2)(1,3)(1,5)(2,1)(2,2)(2,4)(2,6)$ $(3,1)(3,3)(3,5)(3,6)(4,2)(4,4)(4,5)(4,6)$ $(5,1)(5,3)(5,4)(5,5)(6,2)(6,3)(6,4)(6,6)\}$ $\therefore n(A) = 24$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{24}{36} = 0.6667$ <p>A' = neither a multiple of 2 nor a multiple of 3</p> $\therefore P(A') = 1 - P(A) = 1 - 0.667 = 0.3333$ <p>OR</p>	<p>16</p> <p>04</p> <p>1</p> <p>1</p> <p>1</p>



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5.	a)	$\therefore n(S) = 36$ $A =$ neither a multiple of 2 nor a multiple of 3 $A = \{(1,4)(1,6)(2,3)(2,5)(3,2)(3,4)(4,1)(4,3)(5,2)(5,6)(6,1)(6,5)\}$ $\therefore n(A) = 12$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = 0.3333$ OR $n(S) = 36$ $A =$ multiple of 2 $A = \{(1,1)(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)(3,3)(3,5)$ $(4,2)(4,4)(4,6)(5,1)(5,3)(5,5)(6,2)(6,4)(6,6)\}$ $B =$ multiple of 3 $B = \{(1,2)(1,5)(2,1)(2,4)(3,3)(3,6)$ $(4,2)(4,5)(5,1)(5,4)(6,3)(6,6)\}$ $A \cup B =$ multiple of 2 or multiple of 3 $A \cup B = \{(1,1)(1,2)(1,3)(1,5)(2,1)(2,2)(2,4)(2,6)$ $(3,1)(3,3)(3,5)(3,6)(4,2)(4,4)(4,5)(4,6)$ $(5,1)(5,3)(5,4)(5,5)(6,2)(6,3)(6,4)(6,6)\}$ $\therefore n(A \cup B) = 24$ $\therefore p(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{24}{36} = 0.6667$ $(A \cup B)' =$ neither a multiple of 2 nor a multiple of 3 $\therefore P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.6667 = 0.3333$ OR $n(S) = 36$ $A =$ not multiple of 2 $A = \{(1,2)(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4)(3,6)$ $(4,1)(4,3)(4,5)(5,2)(5,4)(5,6)(6,1)(6,3)(6,5)\}$ $B =$ not multiple of 3 $B = \{(1,1)(1,3)(1,4)(1,6)(2,2)(2,3)(2,5)(2,6)(3,1)(3,2)(3,4)(3,5)$ $(4,1)(4,3)(4,4)(4,6)(5,2)(5,3)(5,5)(5,6)(6,1)(6,2)(6,4)(6,5)\}$ $A \cap B =$ neither a multiple of 2 nor a multiple of 3 $A \cap B = \{(1,4)(1,6)(2,3)(2,5)(3,2)(3,4)(4,1)(4,3)(5,2)(5,6)(6,1)(6,5)\}$	<p>1</p> <p>1</p> <p>2</p> <p>1</p> <p>2</p> <p>1</p> <p>1</p>



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5.	a)	$\therefore n(A \cap B) = 12$	2
		$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{12}{36} = 0.3333$	1
		OR	
		$n(S) = 36$	
		$A = \text{multiple of } 2$	
		$A = \{(1,1)(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)(3,3)(3,5)$ $(4,2)(4,4)(4,6)(5,1)(5,3)(5,5)(6,2)(6,4)(6,6)\}$	$\therefore n(A) = 18$
		$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{9}{4}$	1
		$B = \text{multiple of } 3$	
		$B = \{(1,2)(1,5)(2,1)(2,4)(3,3)(3,6)$ $(4,2)(4,5)(5,1)(5,4)(6,3)(6,6)\}$	$\therefore n(B) = 12$
		$\therefore p(B) = \frac{n(B)}{n(S)} = \frac{12}{36} = \frac{2}{6}$	1
$\therefore A \cap B = \{(1,5)(2,4)(3,3)(4,2)(5,1)(6,6)\}$	$\therefore n(A \cap B) = 6$		
$\therefore p(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$	1		
$\therefore p(A \cup B) = p(A) + p(B) - p(A \cap B) = \frac{18}{36} + \frac{12}{36} - \frac{6}{36} = \frac{24}{36} = 0.6667$	$\frac{1}{2}$		
$\therefore p(A \cup B)' = 1 - p(A \cup B) = 1 - \frac{24}{36} = \frac{12}{36} = 0.3333$	$\frac{1}{2}$		
		-----	
	b)	Probability that a bomb dropped from a plane hits a target is 0.4. Two bombs can destroy a bridge, if in all 6 bombs are dropped, find probability that the bridge will be destroyed.	04
	Ans	Given $p = 0.4, n = 6$ and $q = 1 - p = 0.6$	1
		$p(r) = {}^n C_r p^r q^{n-r}$	
		$p(\text{at least two bombs are required to destroy bridge})$	
		$p(r) = 1 - [p(0) + p(1)]$	1
		$= 1 - [{}^6 C_0 (0.4)^0 (0.6)^{6-0} + {}^6 C_1 (0.4)^1 (0.6)^{6-1}]$	1
		$= 0.7667$	1





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5.	c)	If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction. (Given: $e = 2.718$ )	<b>04</b>
	Ans	$p = 0.001, n = 2000$ $\therefore m = np = 0.001 \times 2000 = 2$ $p(\text{more than } 2) = p(3) + p(4) + p(5) + \dots$ $= 1 - [p(0) + p(1) + p(2)]$ $= 1 - \left[ \frac{e^{-2} \cdot (2)^0}{0!} + \frac{e^{-2} \cdot (2)^1}{1!} + \frac{e^{-2} \cdot (2)^2}{2!} \right]$ $= 0.3233$	1 1 1 1
	d)	Evaluate: $\int \frac{\sin(x+a)}{\sin x} dx$	<b>04</b>
Ans	$\int \frac{\sin(x+a)}{\sin x} dx$ $= \int \frac{\sin x \cdot \cos a + \cos x \cdot \sin a}{\sin x} dx$ $= \int \left( \frac{\sin x \cdot \cos a}{\sin x} + \frac{\cos x \cdot \sin a}{\sin x} \right) dx$ $= \int (\cos a + \cot x \cdot \sin a) dx$ $= \cos a \cdot x + \sin a \log(\sin x) + c$	1 1 1 1	
e)	e)	Evaluate: $\int_0^{\pi/2} \log(\sin x) dx$	<b>04</b>
	Ans	Let $I = \int_0^{\pi/2} \log(\sin x) dx$ -----(1)  By property $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$  $I = \int_0^{\pi/4} \log(\sin x) dx + \int_0^{\pi/4} \log\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$  $I = \int_0^{\pi/4} \log(\sin x) dx + \int_0^{\pi/4} \log(\cos x) dx$	$\frac{1}{2}$



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5.	e)	$I = \int_0^{\pi/4} \log(\sin x \cdot \cos x) dx$ $\therefore I = \int_0^{\pi/4} \log\left(\frac{2 \sin x \cdot \cos x}{2}\right) dx$ $\therefore I = \int_0^{\pi/4} \log\left(\frac{\sin 2x}{2}\right) dx$ $\therefore I = \int_0^{\pi/4} \log(\sin 2x) - \log(2) dx$ $\therefore I = \int_0^{\pi/4} \log(\sin 2x) dx - \int_0^{\pi/4} \log(2) dx$ <p>Put <math>2x = t</math></p> $\therefore dx = \frac{dt}{2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>when <math>x \rightarrow 0</math> to <math>\frac{\pi}{4}</math> <math>t \rightarrow 0</math> to <math>\frac{\pi}{2}</math></p> </div> $I = \int_0^{\pi/2} \log(\sin t) \frac{dt}{2} - \log 2 \left[ \frac{\pi}{4} - 0 \right]$ $I = \frac{1}{2} I - \frac{\pi \log 2}{4}$ $\therefore I - \frac{1}{2} I = -\frac{\pi \log 2}{4}$ $\frac{I}{2} = -\frac{\pi \log 2}{4}$ $\therefore I = -\frac{\pi \log 2}{2}$ <p>OR</p> $I = \int_0^{\pi/2} \log(\sin x) dx \quad \text{-----(1)}$ <p>Using Property <math>\int_0^a f(x) dx = \int_0^a f(a-x) dx</math></p> $\therefore I = \int_0^{\pi/2} \log\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>



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5.	e)	$I = \int_0^{\pi/2} \log(\cos x) dx \text{ -----(2)}$ <p>Add (1) and (2)</p> $I + I = \int_0^{\pi/2} \log(\sin x) dx + \int_0^{\pi/2} \log(\cos x) dx$ $2I = \int_0^{\pi/2} \log(\sin x \cdot \cos x) dx$ $\therefore 2I = \int_0^{\pi/2} \log\left(\frac{2 \sin x \cdot \cos x}{2}\right) dx$ $\therefore 2I = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$ $\therefore 2I = \int_0^{\pi/2} (\log(\sin 2x) - \log 2) dx$ $\therefore 2I = \int_0^{\pi/2} \log(\sin 2x) dx - \log 2 \int_0^{\pi/2} dx$ <p>Put <math>2x = t \quad \therefore dx = \frac{dt}{2}</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>when <math>x \rightarrow 0</math> to <math>\frac{\pi}{2}</math> <math>t \rightarrow 0</math> to <math>\pi</math></p> </div> $2I = \int_0^{\pi} \log(\sin t) \frac{dt}{2} - \log 2 \left[ \frac{\pi}{2} - 0 \right]$ $2I = \frac{1}{2} \int_0^{\pi} \log(\sin t) dt - \frac{\pi \log 2}{2}$ $2I = \frac{1}{2} \int_0^{\pi} \log(\sin x) dx - \frac{\pi \log 2}{2}$ $2I = \frac{1}{2} \times 2 \int_0^{\pi/2} \log(\sin x) dx - \frac{\pi \log 2}{2} \quad \text{Using Property}$ $2I = I - \frac{\pi \log 2}{2}$ $\therefore I = -\frac{\pi \log 2}{2}$	<p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p>



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5.	f)	Evaluate: $\int x \sin x \cos x dx$	<b>04</b>
	Ans	$\int x \sin x \cos x dx$ $= \frac{1}{2} \int x 2 \sin x \cos x dx$ $= \frac{1}{2} \int x \sin 2x dx$ $= \frac{1}{2} \left[ x \int \sin 2x dx - \int \left( \int \sin 2x dx \cdot \frac{d}{dx}(x) \right) dx \right]$ $= \frac{1}{2} \left[ x \left( \frac{-\cos 2x}{2} \right) - \int \left( \frac{-\cos 2x}{2} \right) dx \right]$ $= \frac{1}{2} \left[ x \left( \frac{-\cos 2x}{2} \right) + \frac{1}{2} \frac{\sin 2x}{2} \right] + c$ $= \frac{1}{2} \left[ x \left( \frac{-\cos 2x}{2} \right) + \frac{\sin 2x}{4} \right] + c$ <p>OR</p> $\int x \sin x \cos x dx$ <p>Put <math>\sin x = t \Rightarrow x = \sin^{-1} t</math>  <math>\therefore \cos x dx = dt</math></p> $\int \sin^{-1} t \cdot t dt$ $= \sin^{-1} t \int t dt - \int \left( \int t dt \frac{d}{dt}(\sin^{-1} t) \right) dt$ $= \sin^{-1} t \frac{t^2}{2} - \int \frac{t^2}{2} \frac{1}{\sqrt{1-t^2}} dt$ $= \frac{t^2 \sin^{-1} t}{2} - \frac{1}{2} \int \frac{t^2}{\sqrt{1-t^2}} dt$ $= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{2} \int \frac{-t^2}{\sqrt{1-t^2}} dt$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>



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5.	f)	$= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{2} \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt$ $= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{2} \int \left( \frac{1-t^2}{\sqrt{1-t^2}} - \frac{1}{\sqrt{1-t^2}} \right) dt$ $= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{2} \int \left( \sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}} \right) dt$ $= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{2} \left[ \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t - \sin^{-1} t \right] + c$ $= \frac{t^2 \sin^{-1} t}{2} + \frac{1}{4} \left[ t \sqrt{1-t^2} + \sin^{-1} t - 2 \sin^{-1} t \right] + c$ $= \frac{(\sin x)^2 \sin^{-1}(\sin x)}{2} + \frac{1}{4} \left[ (\sin x) \sqrt{1-(\sin x)^2} - \sin^{-1}(\sin x) \right] + c$	<p>1</p> <p>½</p> <p>½</p>
6.	a)	<p><b>Attempt any <u>FOUR</u> of the following:</b></p> <p>It is given that mean and variance of a binomial distribution are 2 and 4/3 respectively what is the probability of obtaining</p> <p>(i) Exactly two successes.</p> <p>(ii) Less than two successes.</p>	<p>16</p> <p>04</p>
	Ans	<p>Given mean = <math>np = 2</math> variance = <math>\frac{4}{3}</math></p> $npq = \frac{4}{3}$ $2q = \frac{4}{3}$ $\therefore q = \frac{2}{3}$ $p = 1 - \frac{2}{3} = \frac{1}{3}$ $\therefore 2 = np$ $\therefore 2 = n \frac{1}{3}$ $\therefore n = 6$ <p>(i) Exactly two successes</p> $\therefore p(2) = {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{6-2}$ $= 0.3292$	<p>½</p> <p>½</p> <p>½</p> <p>1</p>



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6.	a)	(ii) Less than two success $p(0) + p(1) = {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{6-0} + {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1}$ $= 0.3512$	$\frac{1}{2}$ 1
	b)	A card is drawn from a pack of 100 cards numbered 1 to 100 find the probability of drawing a number which is a square. Ans $n(S) = 100$ $A = \text{No. which is square} = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$ $\therefore n(A) = 10$ $\therefore p(A) = \frac{n(A)}{n(S)} = \frac{10}{100} = \frac{1}{10}$ or 0.1	<b>04</b> 1 1 2
	c)	Divide 20 into two parts so that the product of the square of the one and the cube of the other may be the greatest possible. Ans Let two parts of 20 be $x$ and $y$ $\therefore x + y = 20 \Rightarrow y = 20 - x$ $\therefore P = xy$ $P = (20 - x)^2 x^3$ $\therefore P = (400 - 40x + x^2) x^3$ $\therefore P = 400x^3 - 40x^4 + x^5$ $\therefore \frac{dP}{dx} = 1200x^2 - 160x^3 + 5x^4$ $\therefore \frac{d^2P}{dx^2} = 2400x - 480x^2 + 20x^3$ Put $\frac{dP}{dx} = 0$ $\therefore 1200x^2 - 160x^3 + 5x^4 = 0$ $\therefore x^2(1200 - 160x + 5x^2) = 0$ $\therefore x = 0, x = 20, x = 12$ at $x = 12$ $\frac{d^2P}{dx^2} = 2400(12) - 480(12)^2 + 20(12)^3 = -5760 < 0$ Product is greatest when 20 is divided into two parts 12 and 8	<b>04</b>  $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$



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6.	c)	<p>OR</p> <p><math>\therefore x + y = 20 \Rightarrow y = 20 - x</math></p> <p><math>P = xy</math></p> <p><math>P = (20 - x)^3 x^2</math></p> <p><math>\therefore P = (8000 - 1200x + 60x^2 - x^3)x^2</math></p> <p><math>\therefore P = 8000x^2 - 1200x^3 + 60x^4 - x^5</math></p> <p><math>\therefore \frac{dP}{dx} = 16000x - 3600x^2 + 240x^3 - 5x^4</math></p> <p><math>\therefore \frac{d^2P}{dx^2} = 16000 - 7200x + 720x^2 - 20x^3</math></p> <p>Let <math>\frac{dP}{dx} = 0</math></p> <p><math>16000x - 3600x^2 + 240x^3 - 5x^4 = 0</math></p> <p><math>x(16000 - 3600x + 240x^2 - 5x^3) = 0</math></p> <p><math>x = 0, x = 8, x = 20</math></p> <p>at <math>x = 8</math></p> <p><math>\frac{d^2P}{dx^2} = 16000 - 7200(8) + 720(8)^2 - 20(8)^3 = -5760 &lt; 0</math></p> <p>Product is greatest when 20 is divided into two parts 8 and 12</p> <hr/> <p>d) Find the equation of tangent to the curve <math>x = \frac{1}{t}, y = t - \frac{1}{t}</math>, when <math>t = 2</math></p> <p>Ans <math>x = \frac{1}{t}, y = t - \frac{1}{t}</math></p> <p><math>\therefore \frac{dx}{dt} = -\frac{1}{t^2}</math> and <math>\frac{dy}{dt} = 1 + \frac{1}{t^2}</math></p> <p><math>\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + 1/t^2}{-1/t^2} = -(t^2 + 1)</math></p> <p>at <math>t = 2, \frac{dy}{dx} = -5</math></p> <p>at <math>t = 2, x = \frac{1}{2}</math> and <math>y = \frac{3}{2}</math></p> <p>and slope <math>m = -5</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><b>04</b></p> <p><math>\frac{1}{2} + \frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>



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6.	d)	$\therefore$ equation is, $y - y_1 = m(x - x_1)$ $\therefore y - \frac{3}{2} = -5\left(x - \frac{1}{2}\right)$ $\therefore 2y - 3 = -10x + 5$ $\therefore 10x + 2y - 8 = 0$ $\therefore 5x + y - 4 = 0$	1
	e)	<p>Given <math>p(A) = \frac{1}{4}</math> <math>p(B) = \frac{1}{3}</math> and <math>p(A \cup B) = \frac{1}{2}</math></p> <p>Evaluate :</p> <p>(i) <math>p(A/B)</math></p> <p>(ii) <math>p(B/A)</math></p> <p>(iii) <math>p(A \cap B')</math></p> <p>(iv) <math>p(A/B')</math></p> <p>Ans <math>p(A \cup B) = p(A) + p(B) - p(A \cap B)</math></p> $\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - p(A \cap B)$ $\therefore p(A \cap B) = \frac{1}{12}$ <p>(i) <math>p(A/B) = \frac{p(A \cap B)}{p(B)} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}</math></p> <p>(ii) <math>p(B/A) = \frac{p(A \cap B)}{p(A)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{1}{3}</math></p> <p>(iii) <math>p(A \cap B') = p(A) - p(A \cap B) = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}</math></p> <p>(iv) <math>p(A/B') = \frac{p(A \cap B')}{p(B')} = \frac{\frac{1}{6}}{1 - \frac{1}{3}} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}</math></p>	04





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6.	f)	<p>In a certain examination 500 students appeared. Mean score is 68 with S. D. 8. Find the number of students scoring</p> <p>(i) less than 50, (ii) more than 60.</p> <p>Given Area between <math>z = 0</math> to <math>z = 2.25</math> is 0.4878 Area between <math>z = 0</math> to <math>z = 1</math> is 0.3413</p> <p>Ans Given <math>\bar{x} = 68</math> <math>\sigma = 8</math> <math>N = 500</math></p> <p>i) <math>z = \frac{x - \bar{x}}{\sigma} = \frac{50 - 68}{8} = -2.25</math></p> <p><math>\therefore p(\text{Less than } 50) = A(\text{less than } -2.25)</math>  <math>= 0.5 - A(z = 0 \text{ to } z = 2.25)</math>  <math>= 0.5 - 0.4878</math>  <math>= 0.0122</math></p> <p><math>\therefore \text{No. of students} = N \cdot p</math>  <math>= 500 \times 0.0122 = 6.1 \text{ i.e., } 6</math></p> <p>ii) <math>z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 68}{8} = -1</math></p> <p><math>\therefore p(\text{More than } 60) = A(\text{more than } -1)</math>  <math>= A(z = 0 \text{ to } z = 1) + 0.5</math>  <math>= 0.3413 + 0.5</math>  <math>= 0.8413</math></p> <p><math>\therefore \text{No. of students} = N \cdot p = 500 \times 0.8413</math>  <math>= 420.65 \text{ i.e., } 421</math></p>	<p><b>04</b></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
		<p><b><u>Important Note</u></b></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	